

The background of the slide is white, decorated with several large, overlapping circles in four colors: red, teal, lime green, and dark grey. In the center, there is a white rectangular box with a thick lime green border. Inside this box, the text is written in a black, handwritten-style font.

CS 30A: Discussion
Week 6: Balanced Search Tree
Zexi Huang

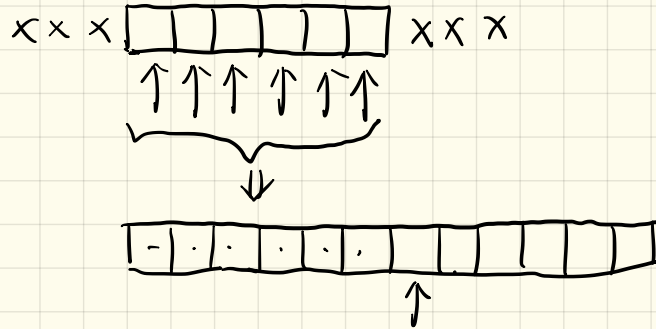
Outlines:

1. Amortized Time
 - Dynamic Array
2. Balanced Search Trees
 - AVL Trees
 - Red-Black Trees
 - Splay Trees
3. Project Assignment 2 Preview

Amortized Time Example: Dynamic Array.

Amortized Time: Average cost for a sequence of operations.

Example: `vector::insert`



① when not full: 1 $O(1)$

② when full: $n+1$ $O(n)$ $\uparrow$

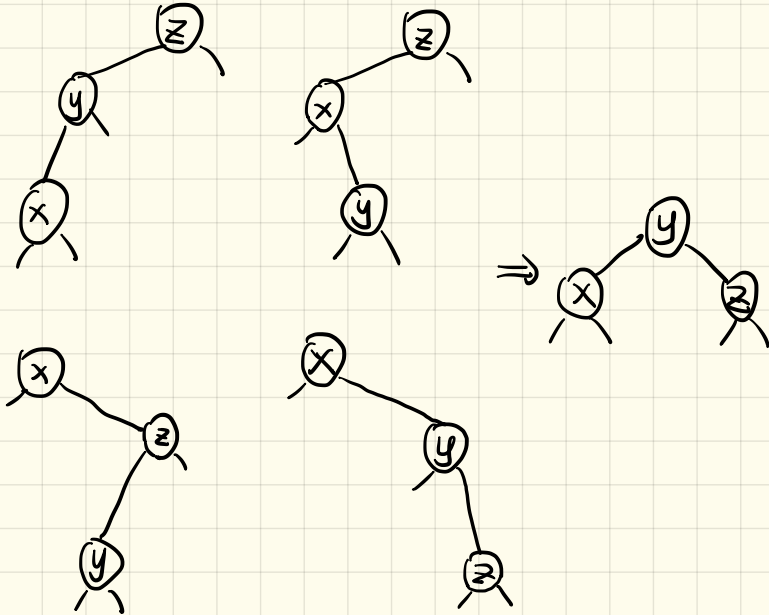
$$\frac{n \cdot 1 + n+1}{n+1} = \frac{2n+1}{n+1} \rightarrow O(1)^* \text{ Amortized}$$

Balanced Search Trees : Comparison

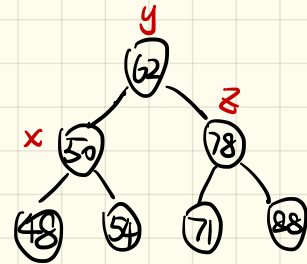
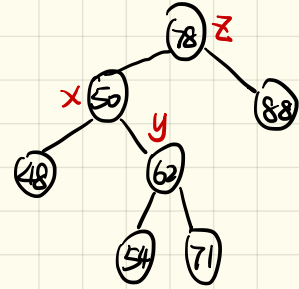
	AVL Tree	Red-Black Tree	Splay Tree
Search	$O(\log n)$	$O(\log n)$	$O(\log n)^* / O(n)$
Insert	$O(\log n)$	$O(\log n)$	$O(\log n)^* / O(n)$
Delete	$O(\log n)$	$O(\log n)$	$O(\log n)^* / O(n)$
Extra Space	Heights of children 2 integers	Red / Black 1 bit	None
Notes	Strictly Balanced Faster Search	Weakly Balanced Faster Insertion/Removal	No Balance Guarantee

Rotations: Trinode Restructuring

Need to rotate x, y, z .



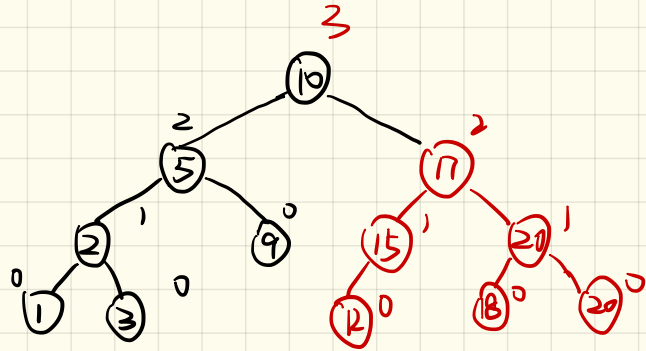
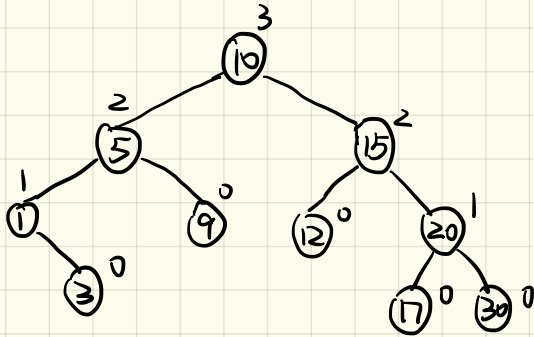
Ex: Rotate 78, 50, 62.



AVL Tree

Insertion: After inserting p , go up from p towards the root, and trinodes are the first node a with unbalanced children, its higher children b , and b 's higher children c .

Ex. Insert 2, 18.

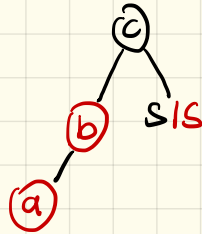
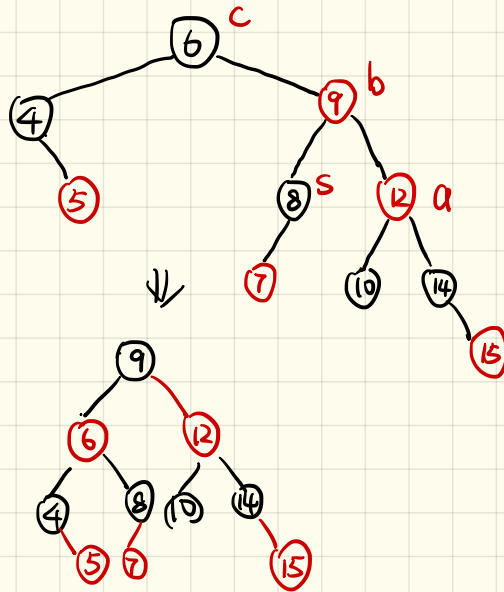
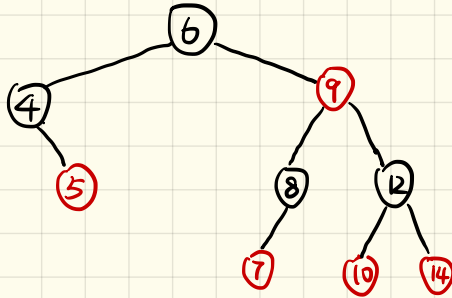


Red-Black Tree

Bottom-up Insertion: Insert a as child of red parent b whose sibling is s and parent c .

- ① If s is black / null, trinodes are a , b and c . Color the new parent black and children red.
- ② If s is red, color b , s black and c red, then deal with c .

Ex. Insert 15.

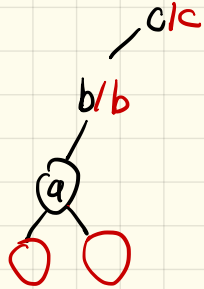
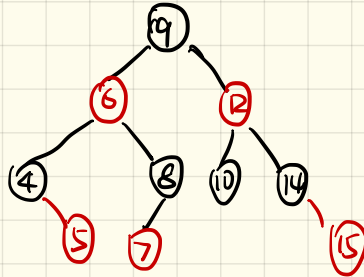
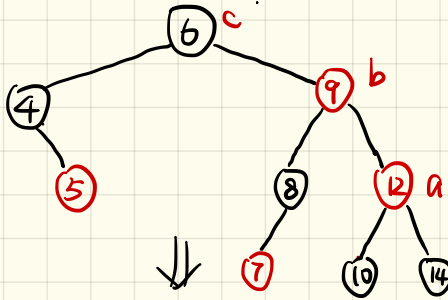
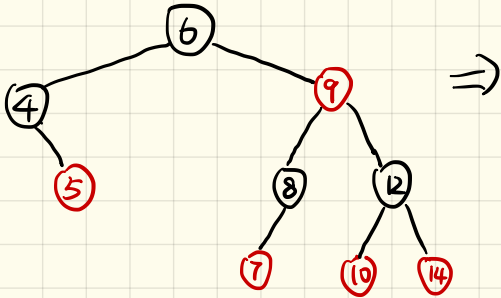


Red-Black Tree

Top-down Insertion: while going down, whenever see node a with two red children, recolor them black, a red, then $\uparrow$ nodes are a , its parent b , and b 's parent c .

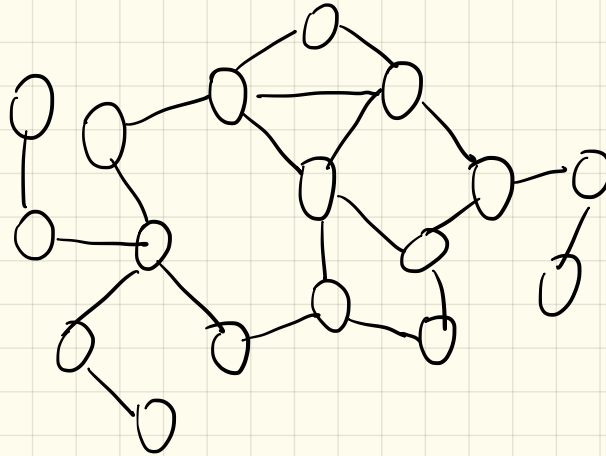
Finally, color the new parent black and children red.

Ex. Insert 15.



PA2 Preview

Graph: V, E



First subquestion: Diameter of the graph?

↑
Maximum of shortest distance between all pairs of nodes.