CMPSC 174A/174N Fundamentals of Database System

Functional Dependencies and Candidate Keys

Discussion Session
Friday, 9:00am-9:50am
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Schedule

- Inferring Functional Dependencies
  - General Algorithm
  - Examples
- Finding Candidate Keys
  - General Algorithm
  - Examples

Inferring Functional Dependencies

- **Problem**
  - For a relation ***R*** and a set of functional dependencies ***F***, infer whether a given FD ***f***=X→Y holds.
  - Example: ***R***=(A,B,C), ***F***={A→B, B→C}. Does ***f***=A→C hold?
- **Algorithm**
  - Find attribute closure of the attributes on the left of the arrow X.
    - *closure*=X.
    - Repeat until there is no change:{If there is an FD U→V where U is a subset of *closure*, add V to *closure*}.
  - If the attributes on the right of the arrow, Y is a subset of *closure*, then ***f*** holds and otherwise not.
- **Is there an FD?**

– Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$

– Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z

– Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

– Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

– Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

Inferring Functional Dependencies

- **Example 1.1**
  - $\boldsymbol{R}$=(A,B,C,D,E), $\boldsymbol{F}$={AB→C, CD→E, DE→B}.
  - Does f=CD→BE hold?
- **Solution**
  - *closure*=CD.
  - CD→E, *closure*=*closure*+E=CDE.
  - DE→B, *closure*=*closure*+B=BCDE.
  - Since BE is a subset of BCDE, f=CD→BE holds.
  - What about f=CD→A?

Inferring Functional Dependencies

- **Example 1.2**
  - $\boldsymbol{R}$=(A,B,C,D,E), $\boldsymbol{F}$={AB→C, CD→E, DE→B}.
  - Does f=ABD→CE hold?
- **Solution**
  - *closure*=ABD.
  - AB→C, *closure*=*closure*+C=ABCD.
  - CD→E, *closure*=*closure*+E=ABCDE.
  - Since CE is a subset of ABCDE, f=ABD→CE holds.
  - Is ABD a superkey?

Inferring Functional Dependencies

- **Example 2.1**
  - $\boldsymbol{R}$=(A,B,C,D,E), $\boldsymbol{F}$={A→BC, CD→E, B→D, E→A, AE→D}.
  - Does f=E→CD hold?
- **Solution**
  - *closure*=E.
  - E→A, *closure*=*closure*+A=AE.
  - A→BC, *closure*=*closure*+BC=ABCE.
  - AE→D, *closure*=*closure*+D=ABCDE.
  - Since CD is a subset of ABCDE, f=E→CD holds.
  - Is E a candidate key?

Inferring Functional Dependencies

- **Example 2.2**
  - $\boldsymbol{R}$=(A,B,C,D,E), $\boldsymbol{F}$={A→BC, CD→E, B→D, E→A, AE→D}.
  - Does f=A→DE hold?
- **Solution**
  - *closure*=A.
  - A→BC, *closure*=*closure*+BC=ABC.
  - B→D, *closure*=*closure*+D=ABCD.
  - CD→E, *closure*=*closure*+E=ABCDE.
  - Since DE is a subset of ABCDE, f=A→DE holds.
  - Is A a candidate key?

Finding Candidate Keys

- **Problem**
  - For a relation ***R*** and a set of functional dependencies ***F***, find all of its candidate keys.
  - Example: ***R***=(A,B,C), ***F***={A→B, B→C}. What is the candidate key?
- **Algorithm**
  - For every subset of all attributes, find its attribute closure.
  - A set of attributes is a candidate key if
    - Its attribute closure contains all the attributes. (It is a superkey)
    - None of its proper subsets' attribute closures contain all the attributes. (It is minimal)
- **Faster Computation**
  - Start from single attributes.
  - Increase the size of attribute set only after checking all attribute sets of previous size.
  - Once an attribute set is found to be a candidate key, all of its proper supersets can't be candidate keys.

Finding Candidate Keys

- **Example 3**
  - $\boldsymbol{R}$=(A,B,C,D), $\boldsymbol{F}$={AB→C, C→D, D→A}.
  - Find all of its candidate keys.
- **Solution**
  - A: *closure*(A)=A
  - B: *closure*(B)=B.
  - C: C→D, D→A, *closure*(C)=ACD.
  - D: D→A, *closure*(D)=AD.
  - AB: AB→C, C→D, *closure*(AB)=ABCD.
  - AC: C→D, *closure*(AC)=ACD.
  - AD: *closure*(AD)=AD.
  - BC: C→D, D→A, *closure*(BC)=ABCD.
  - BD: D→A, AB→C, *closure*(BD)=ABCD.
  - CD: D→A, *closure*(CD)=ACD.
  - Can we stop here?

Finding Candidate Keys

- **Example 4**
  - $\boldsymbol{R}$=(A,B,C,D,E,F,G), $\boldsymbol{F}$={AB→F, AD→E, F→G}.
  - Is ABCD a candidate key?
- **Solution**
  - Attribute closure of ABCD:
    - *closure*=ABCD.
    - AB→F, *closure*=*closure*+F=ABCDF.
    - F→G, *closure*=*closure*+G=ABCDFG.
    - AD→E, *closure*=*closure*+E=ABCDEFG.
  - Attribute closures of subsets:
    - A, B, C, D: *closure*(A)=A, *closure*(B)=B, *closure*(C)=C, *closure*(D)=D.
    - AB, AD: *closure*(AB)=ABFG, *closure*(AD)=ADE.
    - AC, BC, BD, CD: *closure*(AC)=AC, *closure*(BC)=BC, *closure*(BD)=BD, *closure*(CD)=CD.
    - ABC, ACD: *closure*(ABC)=*closure*(AB)+C=ABCFG, *closure*(ACD)= *closure*(AD)+C=ACDE.
    - ABD: *closure*(ABD)=*closure*(AB)+*closure*(AD)=ABDEFG.
    - BCD: *closure*(BCD)=BCD.

Summary

- **Summary**
  - We learnt how to infer FDs and find the keys of a given relation by computing attribute closures.
  - Inferring FDs and finding keys are the cornerstones for determining the level of normal forms for relations.
- **TA Evaluation**
  - Your inputs are important for improving my teaching skills and quality!
  - Written comments are highly appreciated!
    - Please use pen for written comments.
    - If you don't have written comments, please don't write ANYTHING (including name) on the written comment sheet.
- **Q&A**