Is the efficiency of the cycle containing a linear process simple?

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Abstract: The cycle efficiency is an important part in thermodynamics. In most textbooks, the discussions on this topic often focus on the cycles consisting of so-called standard processes. Here a cycle containing a non-standard linear process is investigated. Based on the differential form of the first law of thermodynamics, the conversion point of endothermic and exothermic processes in a linear process is determined and then the thermal efficiencies taking ideal gas and van der Waals gas as the working substances are obtained respectively. These discussions can deepen students' understanding of the contents in thermodynamics.

Keywords: Thermodynamics; cyclic process; linear process; efficiency

1. Introduction

In the teaching of thermodynamics, some key conceptions such as heat, temperature, internal energy, work as well as the first law of thermodynamics which reveals the relations among those physical quantities, are most important but most confusable for many students [1-8]. In many physics textbooks, thermodynamic cycles are introduced based on heat engines. The calculation of the cycle efficiency in a heat engine can show the application of those key conceptions. Most of discussions in the main text [9], however, are limited to the Carnot engine taking ideal gas as the working substance, and the discussions dealing with other heat engines (e.g. Otto engine) are covered in supplementary sections [10, 11]. Meanwhile all the heat engine cycles only consist of standard processes, that is to say, in whole standard process the absorption or the rejection of heat is decided. In Carnot cycle, for example, the absorption of heat only occurs during the isothermal expansion while the rejection of heat only does during isothermal compression.

In this article, a cycle taking ideal gas or van der Waals gas as working substance is composed of a linear expansion process with negative slope, an isobaric compression process, and an isochoric pressurized process in pV diagram (see $X \to Y \to Z \to X$ in Fig.1, called as the linear cyclic process for short), and its efficiency is calculated.

Through the discussion on the these seemingly simple cycle, the reasonable judgment on the endothermic and exothermic processes will be strengthened, the understanding on the differential form of the first law of thermodynamics will be deepened, so as the teaching content in thermodynamics can be grasped correctly for the students.

2. Linear cyclic process for ideal-gas

Firstly, the linear cyclic process with ideal gas as working substance is discussed, as shown in Fig.1. According to the definition of thermal efficiency of heat engine $\eta = 1 - Q_2/Q_1$, Q_1 is the total heat absorbed by the system and Q_2 the total heat rejected by the system respectively.

Note that in Fig.1, the isobaric compression process is exothermic, while the isochoric pressurized process is endothermic, but it is not simple case when it comes to the linear expansion process. When a gas undergoes a linear expansion process, there usually exists a crossover point from endothermic to exothermic process. It is necessary to calculate sectionally the absorption and the release of heat, but not just the algebraic sum of heat in whole process, which is where students often go wrong.

Let $1mol$ ideal gas undergoes a linear expansion process of $p = KV + B$. The differential form of the first law is expressed as

$$\delta Q = \left[(1+i)KV + \left(1 + \frac{i}{2}\right)B \right] dV \qquad (1)$$

where i is the degree of freedom. The crossover point in the linear process can be determined as:

$$V_g = -\frac{i+2}{i+1}\frac{B}{2K} \quad and \quad p_g = \frac{i}{i+1}\frac{B}{2} \qquad (2)$$

It is shown that linear process has an only tangency point $G(p_g, V_g)$ with an adiabatic process. In this way, the absorption and rejection of heat are respectively:

$$Q_{X\to G} = \frac{i}{2}\left(p_g V_g - p_x V_x\right) + \int_{V_x}^{V_g} pdV \quad \text{and} \quad Q_{G\to Y} = \frac{i}{2}\left(p_y V_y - p_g V_g\right) + \int_{V_g}^{V_y} pdV$$

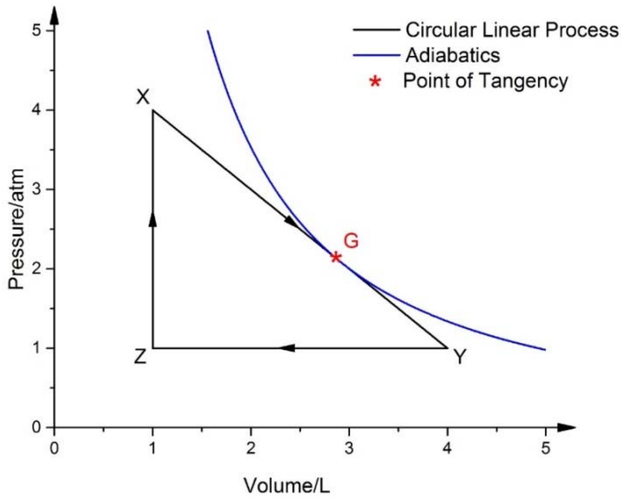

Figure 1. Linear cyclic process with ideal gas as working substance.

The blue curve denotes an adiabatic process tangent with the linear process.

And the absorption and rejection of heat for whole linear cycle are respectively:

$$Q_1 = Q_{X\to G} + Q_{Z\to X} \quad \text{and} \quad Q_2 = Q_{G\to Y} + Q_{Y\to Z}$$

Then the efficiency for the linear cyclic process is

$$\eta = \frac{\left(p_y - p_x\right)^2}{iK\left(p_y V_x - p_g V_g\right) + \left(p_x^2 - p_g^2\right)} \qquad (3)$$

3. Linear cyclic process for van der Waals gas

Based on the equation of state for Van der Waals gas

$$p = \frac{RT}{V-b} - \frac{a}{V^2} \qquad (4)$$

and its internal energy

$$dE = C_v dT + \frac{a}{V^2} dV \qquad (5)$$

the differential form of the first law of thermodynamics in the linear process $X \to Y$ can be expressed as:

$$\delta Q = \{(\frac{2C_v K}{R} + K)V + (\frac{C_v B - C_v bK}{R} + B) + (1 - \frac{C_v}{R})\frac{a}{V^2} + (\frac{2C_v ab}{R})\frac{1}{V^3}\}dV \qquad (6)$$

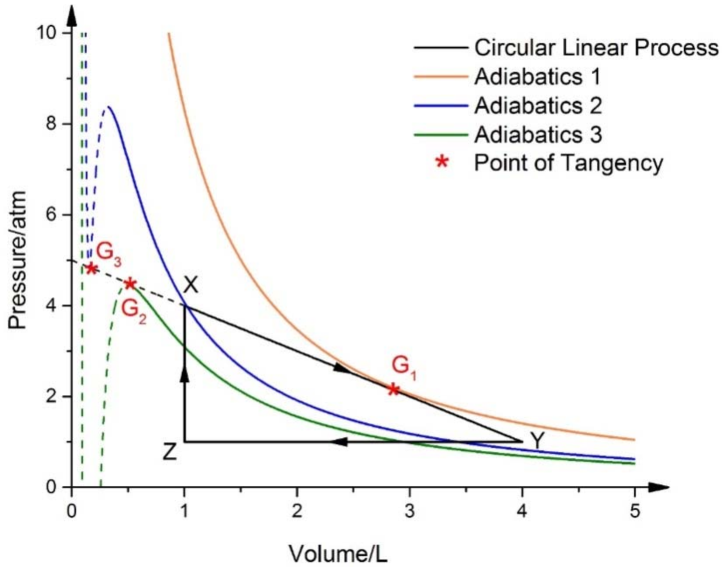

Figure 2. Linear cyclic process taking van der Waals gas as working substance.

The color curves denote three adiabatics tangent with the linear process (and its extension).

Letting $\delta Q = 0$ the crossover point from endothermic to exothermic process can be obtained. It is noteworthy that the linear process may be tangent with the adiabatic family of van der Waals gas on as many as three points mathematically, that is, G_1, G_2, G_3, which are marked with stars in Fig.2. It cannot readily reach, however, the conclusion that there were three crossover points for van der Waals gas. This is because that physically G_3 is in the vapor-liquid transition region while G_2 is very close to the region. Thus, only the tangent point $G_1(p_g, V_g)$ is on the gaseous region, which is similar to the situation for ideal gas. Therefore, the absorption and rejection of heat for the linear process are respectively:

$$Q_{X\to G} = (C_v T_g - \frac{a}{V_g}) - (C_v T_x - \frac{a}{V_x}) + \frac{1}{2}(V_g - V_x)(p_g + p_x) \tag{7}$$

and

$$Q_{G\to Y} = (C_v T_g - \frac{a}{V_g}) - (C_v T_y - \frac{a}{V_y}) - \frac{1}{2}(V_y - V_g)(p_g + p_y) \tag{8}$$

The efficiency for the linear cyclic process is

$$\eta = \frac{(p_y - p_x)^2}{iK(p_y V_x - p_g V_g) + (p_x^2 - p_g^2) - iKab(V_x^{-2} - V_g^{-2}) - Ka(i-2)(V_g^{-1} - V_x^{-1}) - iKb(p_y - p_g)} \tag{9}$$

It is obvious that when $a = b = 0$, the efficiency obtained from Eq.(9) reduces to that from Eq.(3) for ideal gas.

4. Conclusions

This article provides a didactic example in thermodynamics. Through the calculation of efficiency for a linear cyclic process, the conceptions such as absorbed heat, rejected heat and net heat are well demonstrated, which enables students to distinguish endothermic and exothermic processes. Meanwhile it can help students to gain better understanding of the essence and applications of the first law of thermodynamics. Moreover, the comparative analyses on ideal gas and van der Waals Gas shows that although mathematically the characteristics of two models differ enormously, the two models provide similar description of gas with different accuracy and scope, which provides opportunities for students to gain some insight of the relationship between ideal gas and real gas.

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