Learning Node Representations for Information-rich Graphs

Major Area Examination

Zexi Huang

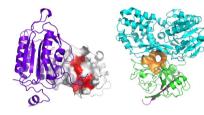
Committee: Ambuj Singh (Chair), Yu-Xiang Wang, Noah Friedkin Department of Computer Science, University of California, Santa Barbara

May 28, 2021

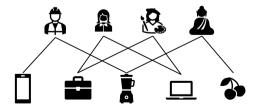
Graphs are ubiquitous



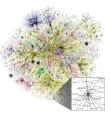
(a) Social network



(b) Protein-protein interactions



(c) Recommendation system



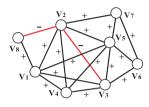
(d) the Internet



(a) Multiscale graph



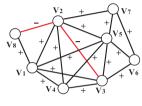
(a) Multiscale graph



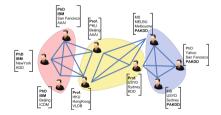
(b) Signed graph



(a) Multiscale graph



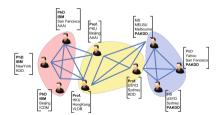
(b) Signed graph

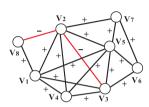


Zexi Huang



(a) Multiscale graph





(b) Signed graph



(d) Physical graph

Social polarization



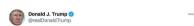


TODAY WE MAKE AMERICA GREAT AGAIN!





The Fake News is working overtime. Just reported that, despite the tremendous success we are having with the economy & all things else, 91% of the Network News about me is negative (Fake). Why do we work so hard in working with the media when it is corrupt? Take away credentials?



These are the things and events that happen when a sacred landslide election victory is so unceremoniously & viciously stripped away from great patriots who have been badly & unfairly treated for so long. Go home with love & in peace. Remember this day forever!

This claim of election fraud is disputed, and this Tweet can't be replied to Retweeted, or liked due to a risk of violence



Replying to @realDonaldTrump

These damned fake news outlets need to be taken down! If they had done this to Ohama, it would've been the end of the world!

We are with you. Mr President!



Replying to @realDonaldTrump

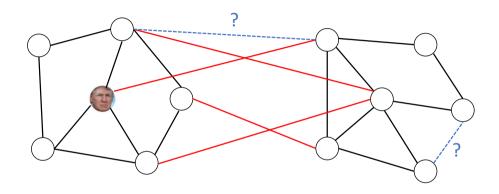
The IDI party will cease to exist once it's all exposed. FAKE NEWS can no longer control [dampen] public awareness of the TRUTH. DARK TO LIGHT.





I love seeing Trump supporters CRY, it's my daily medicine, my weekly energy, my monthly inspiration and my yearly motivation. Their loss is the only reason i'm still alive, i was born to love and enjoy the failure that they have achieved.





Quantifying polarization & signed link prediction

Graph-based applications

- ► (Signed) link prediction
- Node classification

Community detection

Graph regression

Representation learning: extract useful information from data ... for images

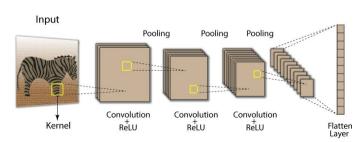
e.g., convolutional neural networks (CNN)

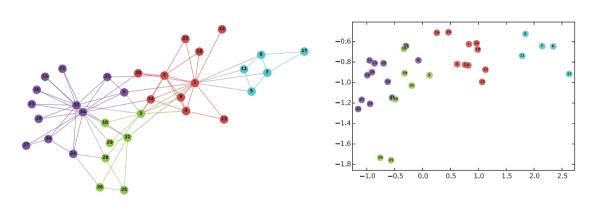
... for text

e.g., skip-gram, transformers

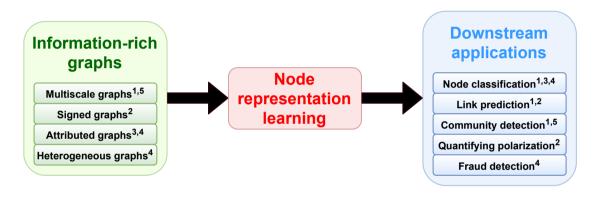
... for graphs

International Conference on Learning Representations (ICLR)





Nodes in a graph $\stackrel{\text{representation learning}}{=\!=\!=\!=\!=\!=}$ feature vectors (embeddings)



¹Huang, Silva, Singh. A broader picture of random-walk based graph embedding. KDD'21.

²Huang, Silva, Singh. Signed embedding for polarized graphs. Working draft.

³Ye, **Huang**, Singh. Krylov graph convolutional networks. Under review.

⁴**Huang**. Graph-based fraud detection in Kindle Direct Publishing. Amazon internship report.

⁵Huang*, Kondapaneni*, Silva, Singh. Multiscale community detection based on PMI. Ongoing.

Outline

- 1. Motivation
- 2. Problems
- 2.1 Random-walk based embedding
- 2.2 Signed graph embedding
- 2.3 Attributed graph embedding
- 3. Conclusions and plan

Earlier work

- Spectral graph theory⁶
- ► Nonlinear dimensionality reduction⁷
- Graph drawing⁸

Deep learning

- Skip-gram for text: word2vec⁹
- ► Skip-gram for graph: DeepWalk¹⁰

⁶Chung. Spectral graph theory. 1997.

⁷Belkin, Niyogi. Laplacian eigenmaps and spectral techniques for embedding and clustering. NeurIPS'02.

⁸Díaz, Petit, Serna. A survey of graph layout problems. CSUR'02.

 $^{^9}$ Mikolov et al. Distributed representations of words and phrases and their compositionality. NeurIPS'13.

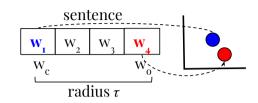
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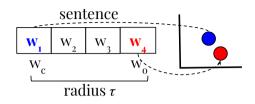
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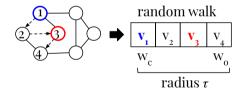
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¹⁰Perozzi, Al-Rfou, Skiena. Deepwalk: Online learning of social representations. KDD'14.

Different random-walks

- ▶ node2vec¹¹: biased towards BFS/DFS
- ► APP¹²: rooted PageRank

Different embedding algorithms

- ► NefMF¹³: explicit matrix factorization
- ► NetSMF¹⁴: spectral sparsifiers for efficiency

Different similarity metrics

- ► Stability¹⁵: autocovariance for multiscale community detection
- ightharpoonup Multiscale 16 : autocovariance embedding as a special case

¹¹Grover, Leskovec. node2vec: Scalable feature learning fornetworks. KDD'16.

¹²Zhou et al. Scalable graph embedding for asymmetric proximity. AAAI'17.

 $^{^{13}}$ Qiu et al. Network embedding as matrix factorization: Unifying deepwalk, line, pte, and node2vec. WSDM'18.

 $^{^{14}}$ Qiu et al. Netsmf: Large-scale network embedding as sparse matrix factorization. WebConf'19.

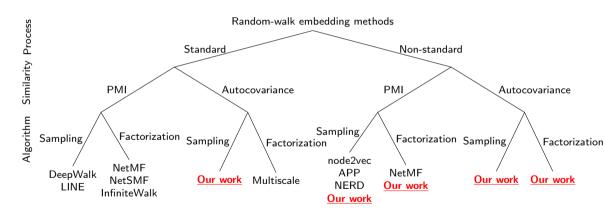
¹⁵Delvenne et al. Stability of graphcommunities across time scales. PNAS'10.

¹⁶Schaub et al. Multiscale dynamical embeddings of complex networks. PRE'19.

Problems & questions:

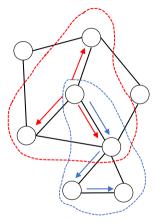
- ▶ Difficult to compare existing methods and to advance the SOTA.
- How should embeddings be used for link prediction?
- How do embeddings capture different structural scales?

A broader picture¹



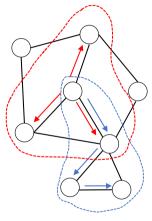
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Our framework

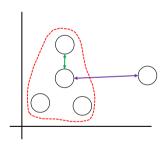


Random-walk process

Our framework

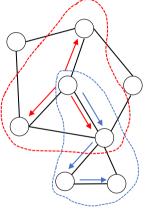


Random-walk process

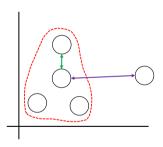


Similarity metric

Our framework



Embedding algorithm



Random-walk process

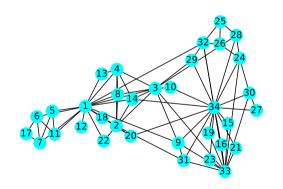
Similarity metric

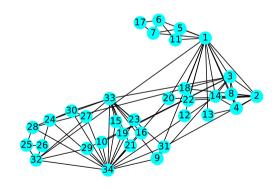
PMI:
$$R = \log(\Pi M^{\tau}) - \log(\pi \pi^{T})$$

Autocovariance: $R = \Pi M^{\tau} - \pi \pi^{T}$

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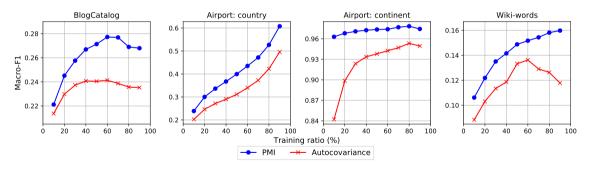


Figure: PMI consistently outperforms autocovariance in node classification.

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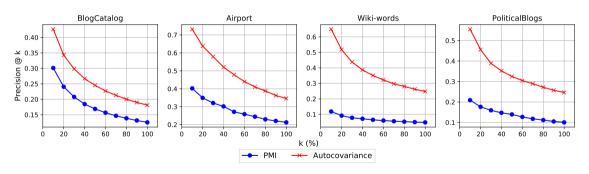


Figure: Autocovariance consistently outperforms PMI in link prediction.

 $\text{predicted degree} \propto \text{embedding norm} \propto \begin{cases} \text{actual degree} & \text{for autocov.} \\ \text{constant} & \text{for PMI} \end{cases}$

Autocovariance captures heterogeneous degree distribution in graphs!

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Autocovariance captures heterogeneous degree distribution in graphs!

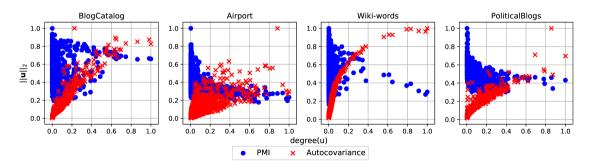


Figure: Autocovariance embedding norms correlated with actual degrees, but not PMI. May 28, 2021 19 / 49

Multiscale

PMI:
$$R = \log(\Pi M^{\tau}) - \log(\pi \pi^{T})$$

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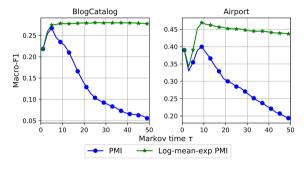


Figure: Node classification performance for PMI can be improved by smooth-averaging across multiple Markov times.

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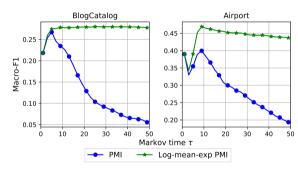


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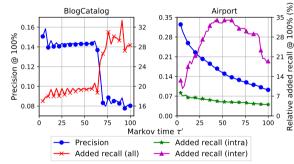


Figure: Prediction of edges of specific structural scales can be improved with different Markov times for autocovariance.

Problems & questions:

- ▶ Difficult to compare existing methods and to advance the SOTA.
- ▶ How should embeddings be used for link prediction?
- ▶ How do embeddings capture different structural scales?

Contributions:

- ► A unified view of different processes, similarities, and algorithms.
- ► Autocovariance embedding is significantly better for link prediction.
- ▶ Ways to exploit multiscale similarity for optimized performance.

PMI-based multiscale community detection⁵

- ➤ Stability^{15,17}: multiscale community detection based on clustered autocovariance
- ► + PMI similarity
- ▶ with Louvain algorithm¹⁸.

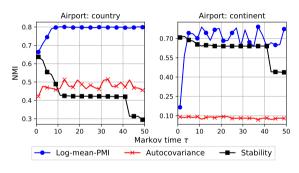


Figure: PMI outperforms both methods based on autocovariance in community detection.

⁵Huang*, Kondapaneni*, Silva, Singh. Multiscale community detection based on PMI. Ongoing.

¹⁵Delvenne et al. Stability of graphcommunities across time scales. PNAS'10.

¹⁷Patelli et al. Generalized Markov stability of network communities. PRE'20.

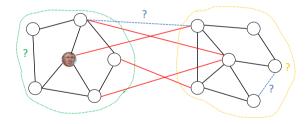
¹⁸Blondel et al. Fast unfolding of communities in large networks. JSTAT'08.

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- 2.2 Signed graph embedding
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Signed graphs: friendly (+) and adversarial (-) relationships

- ▶ e.g., bitcoin markets¹⁹, U.S. Congress²⁰, online social networks²¹
- ► Signed link prediction²²
- ► Polarized community detection²³



¹⁹Kumar et al. Edge weight prediction in weighted signed networks. ICDM'16.

²⁰Thomas, Pang, Lee. Get out the vote: determining support or opposition from congressional floor-debate transcripts. EMNLP'06.

²¹Lai et al. Stance evolution and Twitter interactions in an Italian political debate. NLDB'18.

²²Beigi et al. Signed link prediction with sparse data: The role of personality information. WebConf'19.

²³Bonchi et al. Discovering polarized communities in signed networks. CIKM'19.

Social balance theory²⁴:

- ► SiNE²⁵
- ► SIGNet²⁶
- ► SIDE²⁷

Social status theory²⁸:

► BESIDE²⁹

Network transformation³⁰



²⁴Heider. Attitudes and cognitive organization. J. Psychol.'46.

²⁵Wang et al. Signed network embedding in social media. SDM'17.

²⁶Islam et al. Signet: Scalable embeddings for signed networks. PAKDD'18.

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³⁰ Javari et al. ROSE: Role-based signed network embedding. WebConf'20.

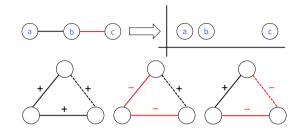
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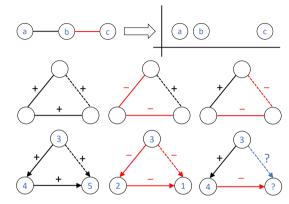
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Existing methods:

- Only consider signed similarity order: positive > negative
- Leave topological similarity order (connected vs disconnected) to unsigned embedding methods

³¹Leskovec et al. Predicting positive and negative links in online social networks. WebConf'10.

³²Chiang et al. Exploiting longer cycles for link prediction in signed networks. CIKM'11.

²²Beigi et al. Signed link prediction with sparse data: The role of personality information. WebConf'19.

Existing methods:

- Only consider signed similarity order: positive > negative
- ► Leave topological similarity order (connected vs disconnected) to unsigned embedding methods
- ▶ Works for edge sign prediction^{31,32} (existence of edges known)
- ▶ In signed link prediction 22 (existence of edges unknown): two-step
 - Apply unsigned methods to predict the existence of edges
 - Predict the signs on those pre-predicted edges

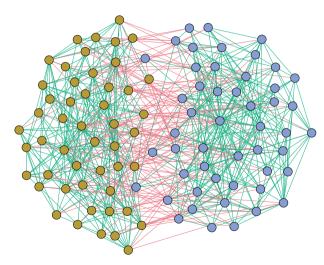
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Signed link prediction in polarized graphs²

- Intra-community edges: dense, positive
- Inter-community edges: sparse, negative (almost impossible to predict for unsigned methods due to sparsity)
- Are real-world signed graphs often polarized?

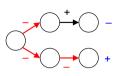


²Huang, Silva, Singh. Signed embedding for polarized graphs. Working draft.

A polarization measure

Unsigned random-walk transition:

$$|M|_{ij} = \sum_{\mathrm{all\ paths}\ l\ \mathrm{between}\ i\ \mathrm{and}\ j} \mathsf{Prob}(l)$$



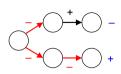
Signed random-walk transition:

$$M_{ij} = \sum_{\text{all paths } l \text{ between } i \text{ and } j} \mathsf{Prob}(l) \mathsf{Sign}(l) \leftarrow \mathsf{from balance theory}$$

A polarization measure

Unsigned random-walk transition:

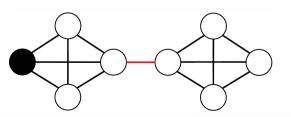
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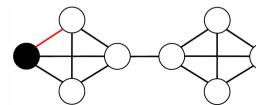


Signed random-walk transition:

$$M_{ij} = \sum_{\mathsf{all\ paths}\ l\ \mathsf{between}\ i\ \mathsf{and}\ j} \mathsf{Prob}(l) \mathsf{Sign}(l) \leftarrow \mathsf{from\ balance\ theory}$$

- ▶ Key observation: $|M|_{:i}$ and $M_{:i}$ are highly correlated if i is polarized.
- ▶ Define $Pol(i) = corr(|M|_{i}, M_{i})$, and Pol(G) = average of <math>Pol(i).





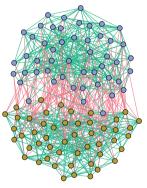
Least polarized politicians in U.S. Congress

- ▶ Henry Cuellar (Pol = -0.65), Democrat, "moderate-centrist", voted with President Trump 75% of the time³³
- ▶ Jane Harman (Pol = -0.54), Democrat, "centralist", "the best Republican in the Democratic Party" ³⁴

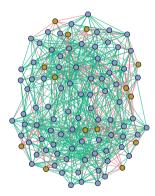
³³Malone. A Q&A with the House Democrat who's voted with trump 75 percent of the time. FiveThirtyEight'17.

³⁴Skelton. California and the west: in the ring, with contenders for governor. LA Times'98.

Polarization of real-world graphs



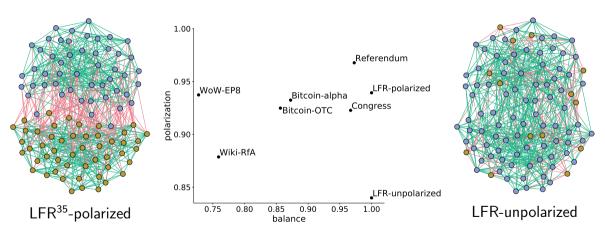
LFR³⁵-polarized



LFR-unpolarized

³⁵Lancichinetti et al. Benchmark graphs for testing community detection algorithms. PRE'08.

Polarization of real-world graphs



³⁵Lancichinetti et al. Benchmark graphs for testing community detection algorithms. PRE'08.

Signed random-walk similarity

- Existing methods: keep signed similarity order (positive > negative)
- Proposed model: extend autocovariance for signed random-walks with full similarity order (positive > disconnected > negative)
- Predict polarized inter-community edges with most dissimilar pairs

Signed random-walk similarity

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- Proposed model: extend autocovariance for signed random-walks with full similarity order (positive > disconnected > negative)
- Predict polarized inter-community edges with most dissimilar pairs

Table: Average dot product similarity between embeddings of node pairs.

	Wiki-RfA	Bitcoin-OTC	Bitcoin-alpha	Congress	WoW-EP8	Referendum
Positive	13.11	17.02	18.09	21.54	1.63	4.62
Disconnected	-0.01	-0.01	-0.02	-0.28	-0.24	-0.01
Negative	-5.92	-70.95	-56.82	-27.59	-2.85	-9.47

Signed link prediction

- ► For our method: rank signed similarity and predict positive/negative edges as the most similar/dissimilar pairs
- ► For baselines: train two classifiers on the concatenated node embeddings for positive and negative edges separately
- Knowledge on the existence of edges unavailable

Signed link prediction

Table: Our method significantly outperforms baselines in signed link prediction.

Precision @ 100%		SIGNet	SIDE	BESIDE	SiNE	ROSE	Our work
Wiki-RfA	Positive Negative	0.0028 0.0000	0.0002 0.0043	0.0056 0.0016	0.0098 0.0007	0.0379 0.0132	0.1795 0.0112
Bitcoin-OTC	Positive	0.0008	0.0003	0.0325	0.0058	0.0494	0.0935
	Negative	0.0000	0.0045	0.0076	0.0030	0.0227	0.2727
Bitcoin-alpha	Positive	0.0000	0.0004	0.0259	0.0040	0.0626	0.1181
	Negative	0.0000	0.0032	0.0226	0.0097	0.0226	0.0903
Congress	Positive	0.0000	0.0000	0.0125	0.0000	0.0250	0.1250
	Negative	0.0000	0.0000	0.0000	0.0400	0.0000	0.1600
WoW-EP8	Positive	0.2104	0.0949	0.1348	0.1831	0.4060	0.6201
	Negative	0.0361	0.0749	0.0553	0.0395	0.1221	0.1475
Referendum	Positive	0.0002	0.0465	0.0413	0.0440	0.0854	0.1900
	Negative	0.0000	0.0020	0.0056	0.0000	0.0234	0.0429

Signed link prediction with unsigned link prediction knowledge

- Combine the ranking scores from each method with unsigned reconstructed autocovariance similarity
- Train two classifiers to predict positive/negative edges with combined scores
- Knowledge on the existence of edges available

Signed link prediction with unsigned link prediction knowledge

Table: Our method outperforms baselines, especially for (polarized) negative edges.

Precision @ 100%		SIGNet	SIDE	BESIDE	SiNE	ROSE	Our work
Wiki-RfA	Positive	0.1700	0.1694	0.1722	0.1726	0.1710	0.1810
	Negative	0.0290	0.0289	0.0308	0.0314	0.0335	0.0613
Bitcoin-OTC	Positive	0.0811	0.0824	0.0882	0.0805	0.0844	0.0954
	Negative	0.1939	0.2030	0.2242	0.2152	0.1894	0.2848
Bitcoin-alpha	Positive	0.1053	0.1113	0.0994	0.1018	0.1145	0.1157
	Negative	0.1194	0.1226	0.1194	0.1161	0.1065	0.1387
Congress	Positive Negative	0.1000 0.0400	0.1125 0.0400	0.0875 0.0000	0.1000 0.0000	0.1375 0.0000	0.1000 0.1600
WoW-EP8	Positive	0.4454	0.4624	0.4553	0.4605	0.6014	0.6163
	Negative	0.0911	0.1367	0.1288	0.1233	0.1644	0.2539
Referendum	Positive	0.1746	0.1775	0.1719	0.1706	0.1714	0.1903
	Negative	0.0461	0.0326	0.0942	0.0679	0.0457	0.1132

Contributions

- ► Design a polarization measure for nodes in/and signed graphs.
- ▶ Identify the problem of past work in polarized signed link prediction.
- Propose a signed embedding model that solves the problem and significantly outperforms SOTA in various real-world signed graphs.

Future work

- Extending other types of similarity to signed graphs (e.g., PMI).
- ► A model that leverages the polarization measure to guide the use of signed and unsigned similarity for signed link prediction.

Outline

- 1. Motivation
- 2. Problems
- 2.1 Random-walk based embedding
- 2.2 Signed graph embedding
- 2.3 Attributed graph embedding
- Conclusions and plan

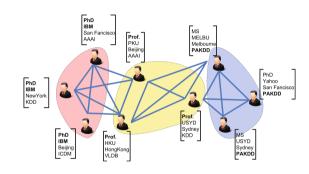
Problems

Attributed graphs

- Citation networks³⁶. e.g., Cora, Pubmed
- ► Wikipedia networks³⁷: e.g., Chameleon, Squirrel

Embedding objectives

- ► Unsupervised³⁸: e.g., topological similarity
- ► (Semi-)supervised³⁹: e.g., node classification



³⁶Sen et al. Collective classification in network data. Al Magazine'08.

³⁷Rozemberczki et al. Multi-scale attributed node embedding. J. Complex Netw.'21.

³⁸Kipf, Welling. Variational graph auto-encoders. arXiv'16.

³⁹Kipf, Welling. Semi-supervised classification with graph convolutional networks. arXiv'16.

Graph Neural Networks (GNN)

Rationale: learn to aggregate attributes from neighbors.

Aggregation

► GCN³⁹, GraphSAGE⁴⁰

Edge weighting

► AGNN⁴¹, GAT⁴², GraphSAGE

Multi-hop neighborhood

► LanczosNet⁴³, PPNP⁴⁴

TARGET NODE

³⁹Kipf, Welling. Semi-supervised classification with graph convolutional networks. arXiv'16.

⁴⁰Hamilton et al. Inductive representation learning on large graphs. NeurIPS'17.

⁴¹Thekumparampil et al. Attention-based graph neural network for semi-supervised learning. arXiv'18.

⁴²Veličković et al. Graph attention networks. ICLR'18.

⁴³Liao et al. Lanczosnet: Multi-scale deep graph convolutional networks. ICLR'19.

⁴⁴Klicpera et al. Predict then propagate: Graph neural networks meet personalized pagerank. ICLR'19.

Challenges:

- Stacking too few layers can't capture the multi-hop information.
- Stacking too many layers leads to oversmoothing^{45,46}.

Solution: learn to aggregate multi-hop neighborhood³

 $^{^{45}}$ Li et al. Deeper insights into graph convolutional networks for semi-supervised learning. AAAI $^{\prime}18$

⁴⁶Xu et al. Representation learning on graphs with jumping knowledge networks. ICML'18.

³Ye, **Huang**, Singh. Krylov graph convolutional networks. Under review.

Simple Graph Convolution⁴⁷:

$$H^{(k)} = \sigma(\widetilde{M}^k X \Theta)$$

 $H^{(k)}$: kth order embedding, X: node attributes, Θ : trainable weights \widetilde{M} : random-walk transition with added self-loops

⁴⁷Wu et al. Simplifying graph convolutional networks. ICML'19.

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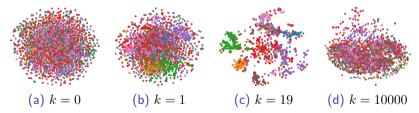


Figure: Visualization of embedding without training for Cora. Colors denote actual classes. A good choice of neighborhood helps reveal the cluster structure.

⁴⁷Wu et al. Simplifying graph convolutional networks. ICML'19.

Weighting neighborhoods

- ▶ Need to assign right weights to different orders of neighborhoods.
- Crucial for semi-supervised learning in sparsely labeled graphs.
- Weights should be adaptive to different datasets.

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Krylov Graph Convolutional Network

$$H = \sigma(\sum_{k=0}^{K} \alpha_k \widetilde{M}^k X \Theta)$$

- ▶ Weights for different neighborhoods are learned for each dataset.
- ► Krylov tensor $(X\Theta, \widetilde{M}X\Theta, ..., \widetilde{M}^KX\Theta)$ can be computed iteratively for efficiency, since \widetilde{M} is sparse.
- ightharpoonup This procedure can also be viewed as the power iteration for \widetilde{M} .

Semi-supervised node classification for sparsely labeled graphs

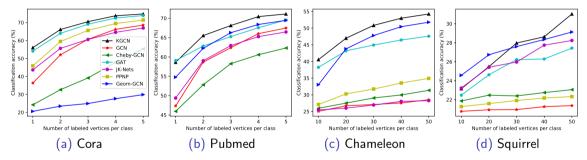


Figure: Our method (KGCN) outperforms the baselines in almost all cases.

Learned neighborhood weights

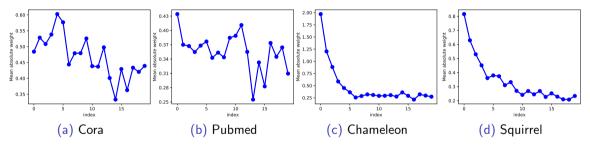


Figure: Mean neighborhood weights (over 30 splits) are different across datasets. Note that Cora and Pubmed have larger weights for high-order neighborhoods compared to Chameleon and Squirrel.

Contributions:

- ► Interpret the GCN propagation rule as power iteration, with its Krylov tensor encoding multi-hop neighborhood information.
- ▶ Propose to learn the weights for combining different orders of neighborhood information, adaptive to different datasets.
- Outperforms SOTA on various sparsely labeled graphs.

Leveraging topology and attribute information

- ► Internship: attributed multiplex heterogeneous graph embedding⁴⁸
 - ► GCNs, GATs, random-walks, skip-grams, etc.
 - ▶ Authors provide a over-simplified implementation.

Zexi Huang

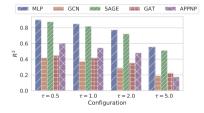
⁴⁸Cen et al. Representation learning for attributed multiplex heterogeneous network. KDD'19.

⁴⁹Ma et al. CopulaGNN: Towards integrating representational and correlational roles of graphs in GNNs. ICLR'21.

⁵⁰Srinivasan, Ribeiro. On the equivalence between node embeddings and structural graph representations. ICLR'20.

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All GNNs underperform the basic MLP in regression⁴⁹.

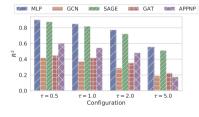
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All GNNs underperform the basic MLP in regression⁴⁹.

- Need better ways to use topological information in GNNs, not just as the computational graph for aggregation.
- As an attempt, CopulaGNN⁴⁹ adds correlational roles for node-level regression.
- Topological information is even more important for link prediction⁵⁰.

⁴⁸Cen et al. Representation learning for attributed multiplex heterogeneous network. KDD'19.

⁴⁹Ma et al. CopulaGNN: Towards integrating representational and correlational roles of graphs in GNNs. ICLR'21.

⁵⁰Srinivasan, Ribeiro. On the equivalence between node embeddings and structural graph representations. ICLR'20.

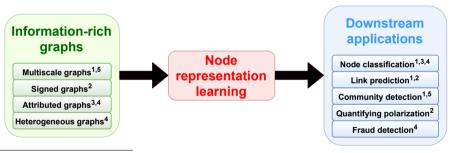
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Conclusions: extracting key information for applications

- ► Heterogeneous degree distribution for link prediction¹
- ► Full similarity order for polarized signed link prediction²
- ► Adaptive neighborhood weighting for sparsely labeled node classification³



¹Huang, Silva, Singh. A broader picture of random-walk based graph embedding. KDD'21.

²Huang, Silva, Singh. Signed embedding for polarized graphs. Working draft.

³Ye, **Huang**, Singh. Krylov graph convolutional networks. Under review.

⁴Huang. Graph-based fraud detection in Kindle Direct Publishing. Amazon internship report.

⁵Huang*, Kondapaneni*, Silva, Singh. Multiscale community detection based on PMI. Ongoing.

Plan

Jun/21: Start working on leveraging topology and attributes

Aug/21: Finish paper on signed embedding

Oct/21: Finish paper on PMI clustering

Jun/22: Proposal

• • •

?/23: Defense