

Learning Representations for Information-rich Graphs

PhD Defense

Zexi Huang

Committee: Ambuj Singh (Chair), Yu-Xiang Wang, Xifeng Yan

Department of Computer Science, University of California, Santa Barbara

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Representation learning

“Extracting key information from raw data to enable effective data science applications via deep neural networks.”

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Representation learning

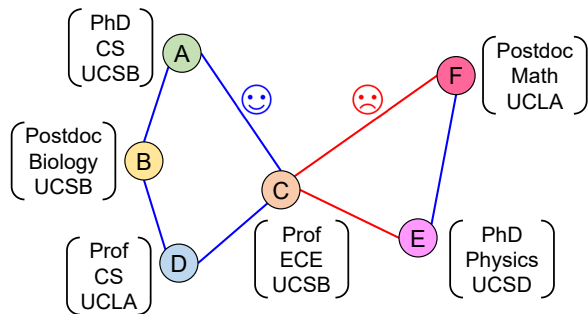
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- ▶ For images: Convolutional Neural Networks (CNNs)
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- ▶ **For graphs**

Graph applications

- ▶ Social network analysis: community detection
- ▶ Product and video recommendation: link prediction
- ▶ Financial fraud detection: anomaly detection
- ▶ Novel drug discovery: graph classification

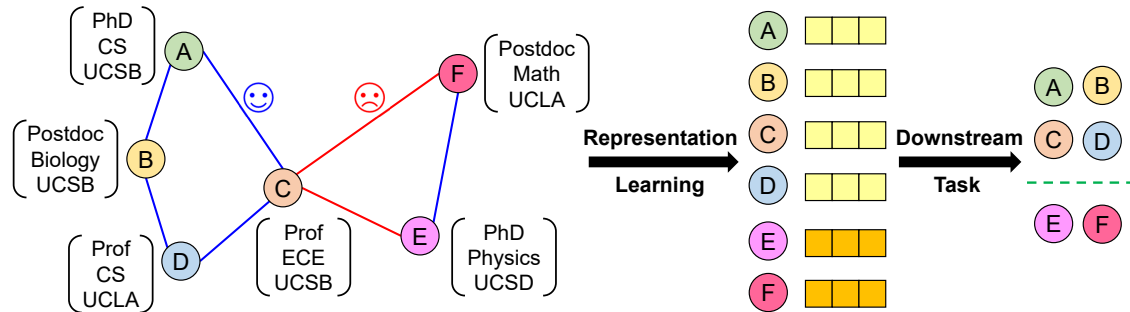
Representation learning for information-rich graphs



Graph structure: *relationship information*

Node and edge attributes: *descriptive information*

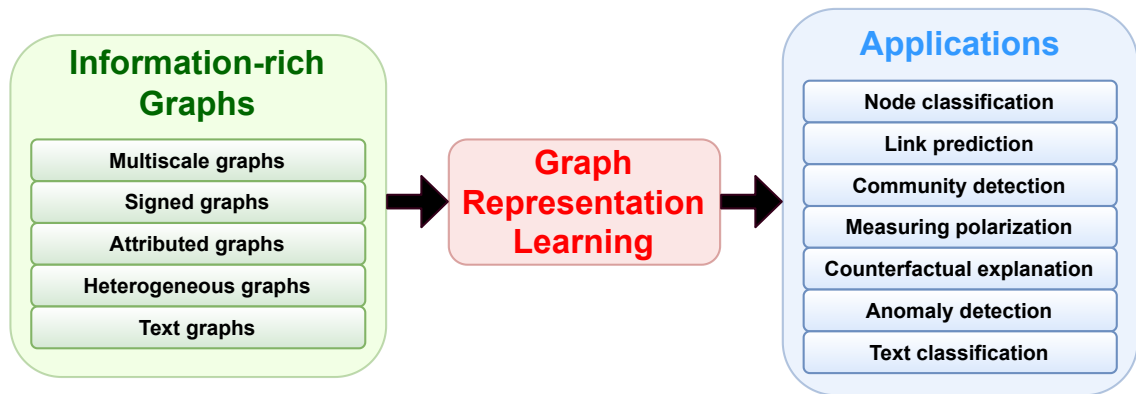
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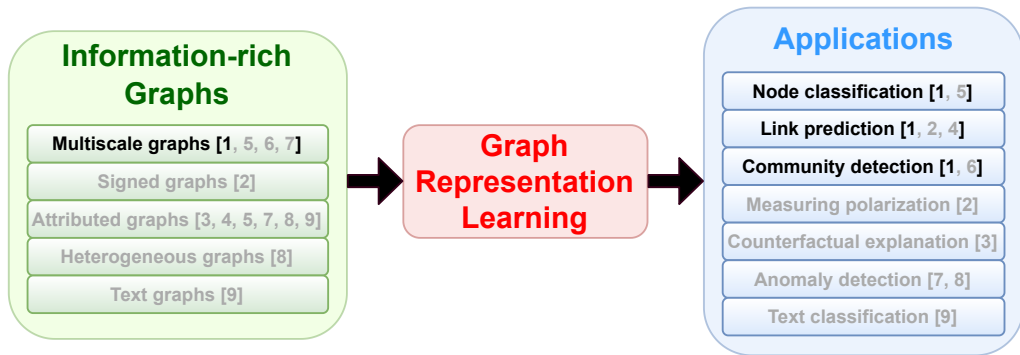


Graph structure: *relationship information*

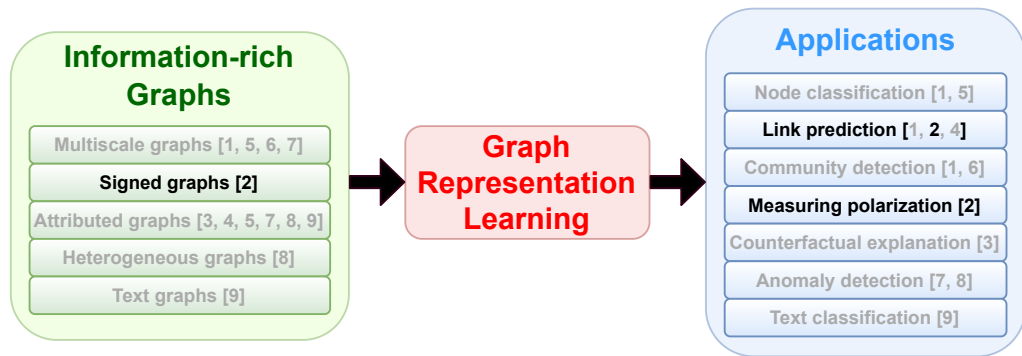
Node and edge attributes: *descriptive information*

My research



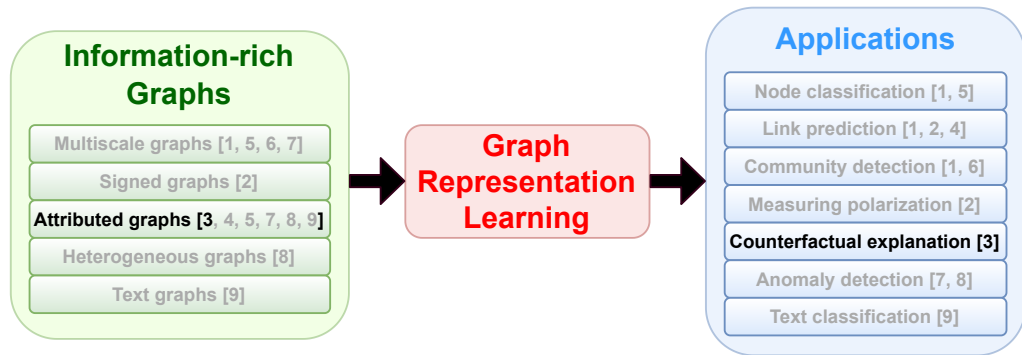


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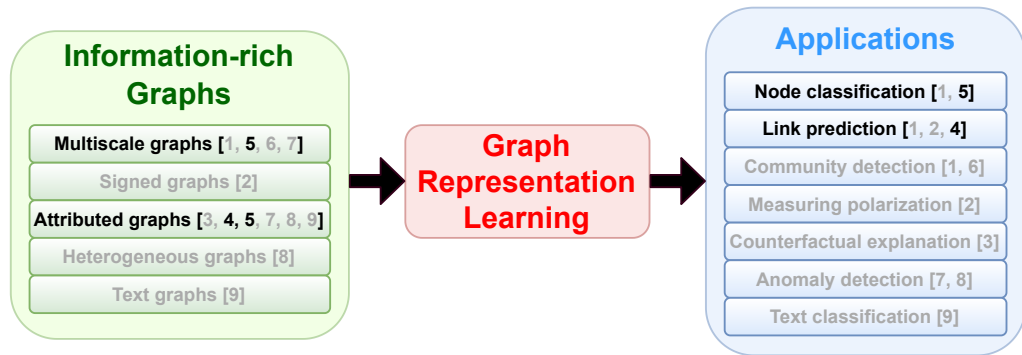
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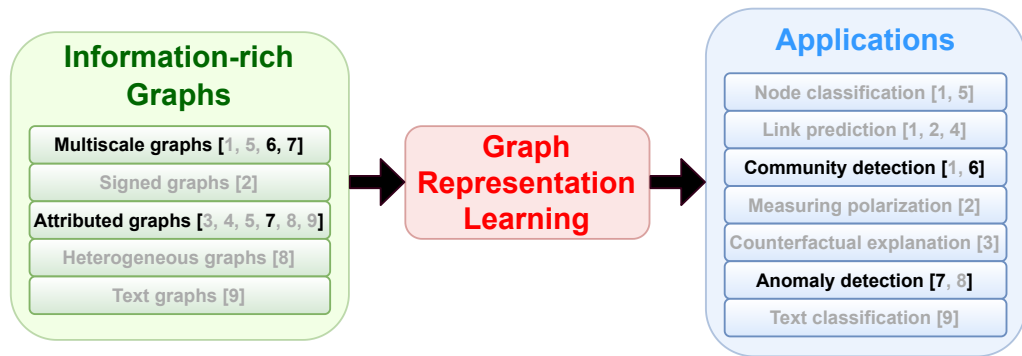
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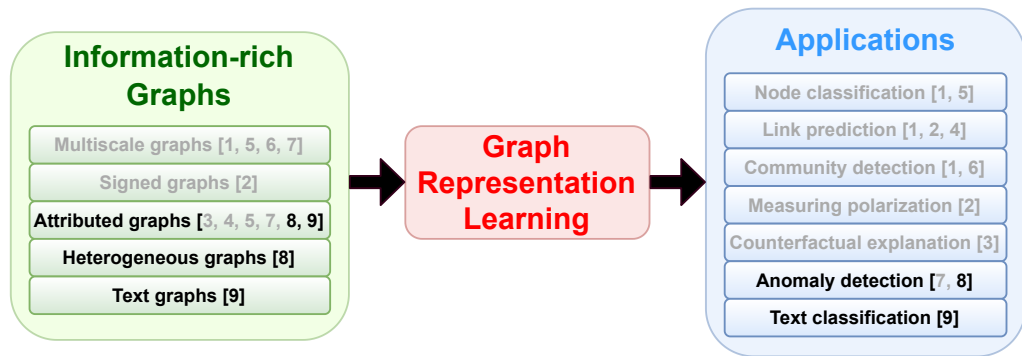
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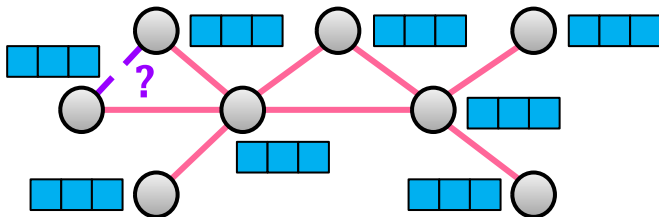
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Outline

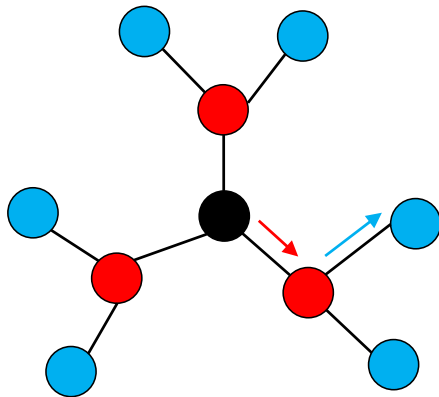
1. Introduction
2. Graph representation learning for link prediction
3. Graph representation learning for graph classification
4. Graph representation learning for tasks on multiscale graphs
5. Conclusion and future work

Link prediction

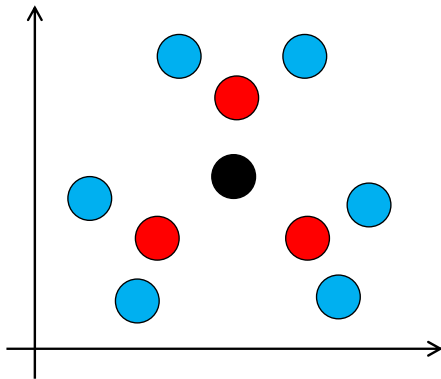


Link prediction: predicting the unobserved interactions (edges) between nodes given the observed graph structure (topology) and other information (e.g., node attributes).

Random-walk based node embedding

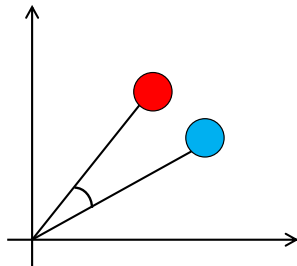


(a) Use random-walks to capture topological proximity (similarity)

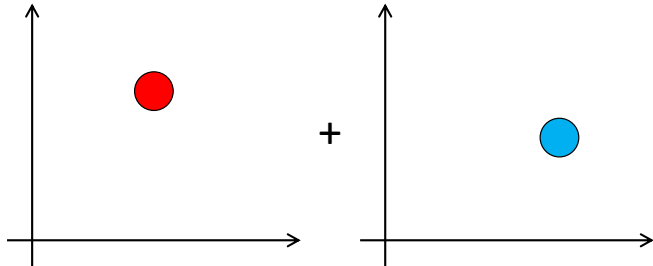


(b) Find low-dimensional latent vectors (embeddings) to preserve similarity

Link prediction based on node embedding

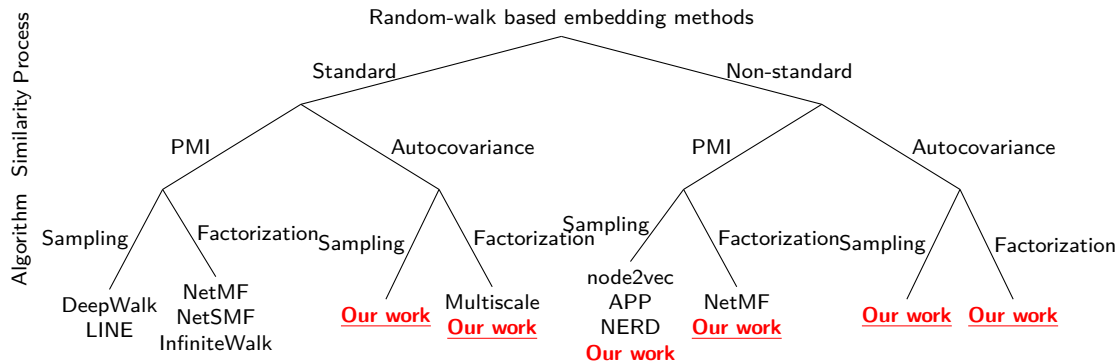


(a) Dot product [1, 2, 3]



(b) Classification based on combined embeddings [4, 5, 6]

A broader picture of random-walk based embedding¹



- ▶ We present a unified framework for random-walk based embedding.
- ▶ We find that Autocovariance enables state-of-the-art link prediction.

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Random-walk based similarity metrics

M : transition matrix π , Π : stationary distribution τ : random-walk scale

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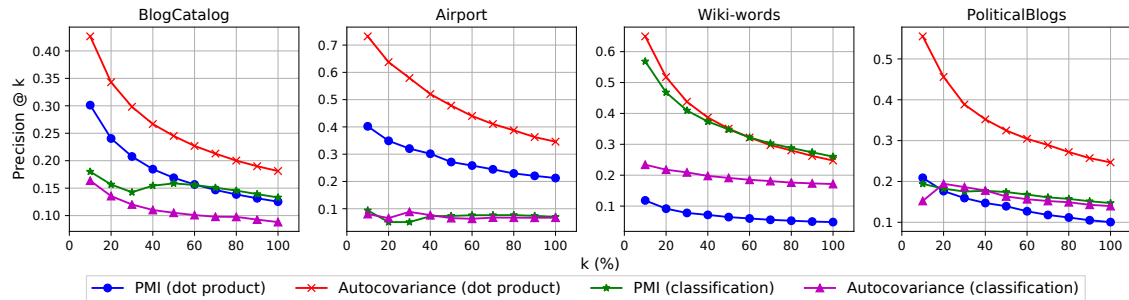


Figure: Autocovariance with dot product ranking consistently outperforms PMI (with either ranking scheme) in link prediction.

Understanding the difference

$$\text{predicted degree} \propto \text{embedding norm } \|\mathbf{u}\| \propto \begin{cases} \text{actual degree} & \text{for AC} \\ \text{constant} & \text{for PMI} \end{cases}$$

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Autocovariance captures **heterogeneous degree distribution** in graphs!

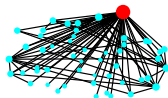
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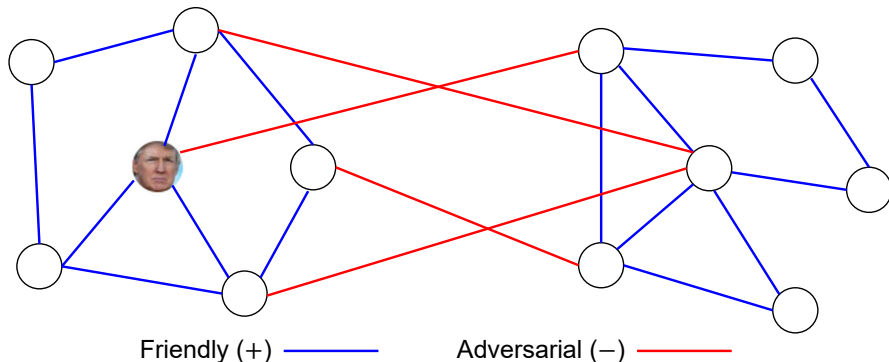
(a) PMI



(b) Autocovariance

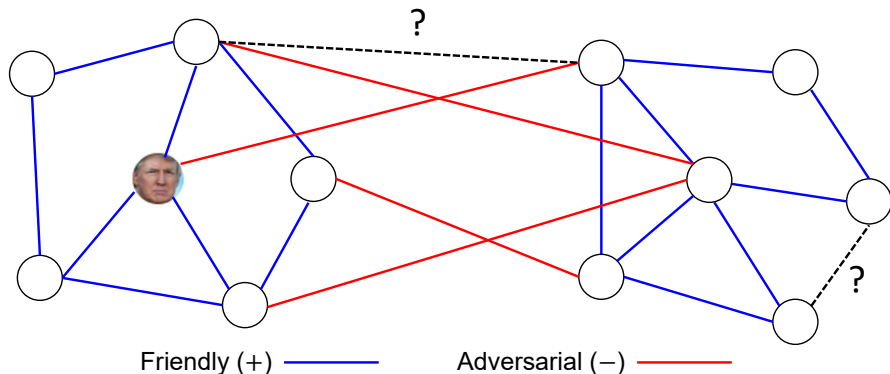
Figure: Autocovariance predicts more edges connecting to the hubs than PMI.

Polarized embedding for effective signed link prediction²



²Huang, Silva, Singh. POLE: Polarized embedding for signed networks. WSDM'22.

Polarized embedding for effective signed link prediction²



- ▶ We identify the key challenge of embedding polarized signed graphs.
- ▶ We develop polarized embedding for SOTA negative link prediction.

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Signed link prediction in polarized networks

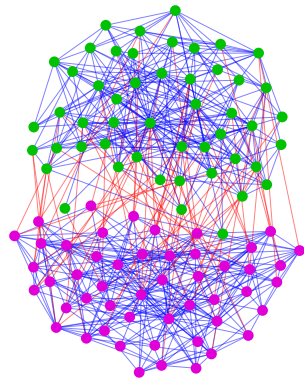


Figure: A synthetic polarized graph based on the LFR benchmark [7].

Signed link prediction in polarized networks

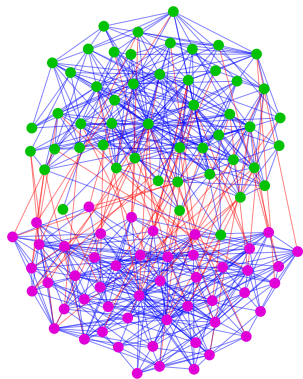


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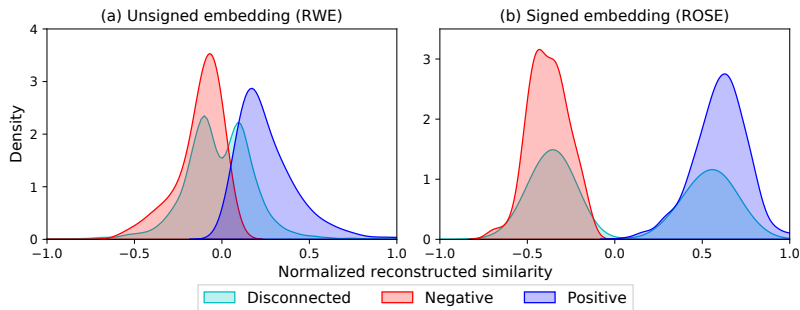


Figure: Distributions of the reconstructed similarity for different types of node pairs in the polarized graph using (a) unsigned [8] and (b) signed [6] embedding.

POLE: polarized embedding

- ▶ Signed random-walks to capture both similarities:

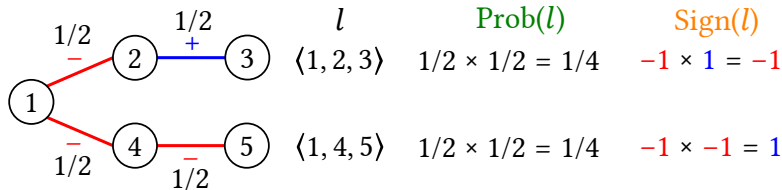
$$M_{uv}(t) = \sum_{\text{all length-}t \text{ paths } l \text{ between } u \text{ and } v} \text{Prob}(l) \text{Sign}(l)$$

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- ▶ $\text{Prob}(l)$ captures the unsigned similarity.
- ▶ $\text{Sign}(l)$ based on the social balance theory captures the signed similarity.

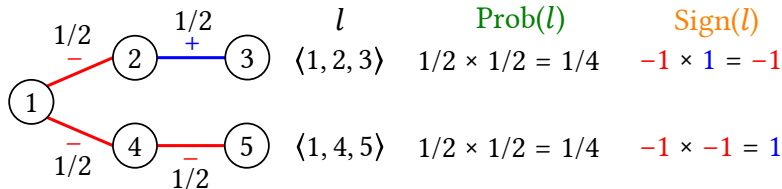


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- ▶ POLE: extends Autocovariance similarity [9, 8] to signed RW

$$R(t) = M(t)^{\top} W M(t)$$

Polarized similarity consistency

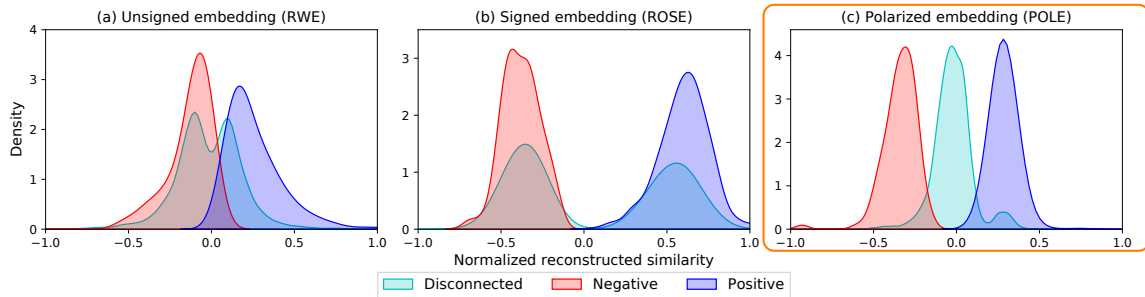


Figure: Polarized embedding (c) preserves *polarized similarity consistency*—negative pairs are separated from other pairs in the similarity spectrum—while unsigned embedding (a) and signed embedding (b) fail to do so.

Signed link prediction results

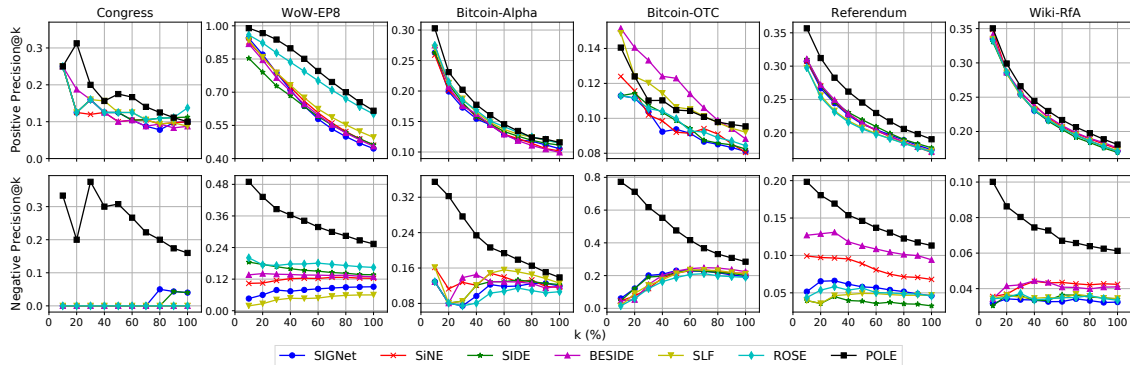
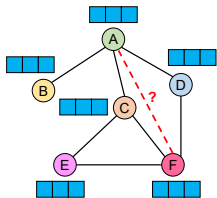


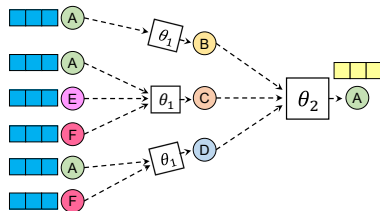
Figure: Signed link prediction with link existence information performance comparison. POLE outperforms all baselines in almost all datasets, especially for the negative links.

Attributed graph embedding and graph neural networks

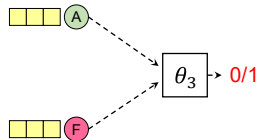
- ▶ GNNs [10, 11, 12] are a powerful DL paradigm that learns to generate better node features (embeddings) using structure information.



(a) Input attributed graph



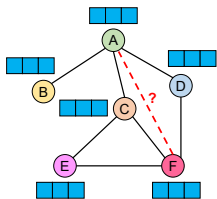
(b) GNN message-passing



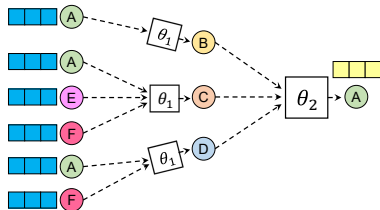
(c) Pooling and classification

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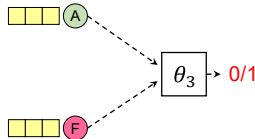
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- ▶ Advantages over topological heuristics for link prediction:
 - ▶ Potential to discover new heuristics via **supervised learning**.
 - ▶ Natural incorporation of **node attribute** information.

Challenges of GNNs for link prediction

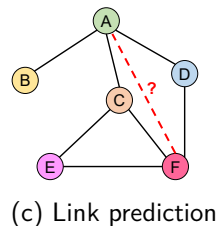
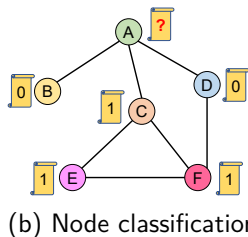
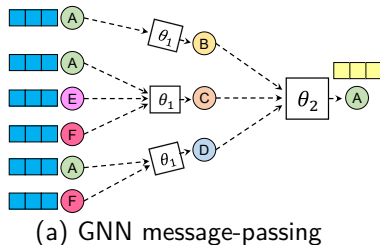


Figure: The attribute-centric message-passing mechanism is effective for tasks **on** the topology, e.g., node classification. Link prediction, however, is a task **for** the topology.

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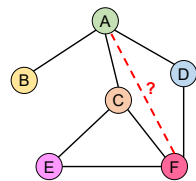
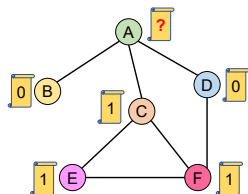
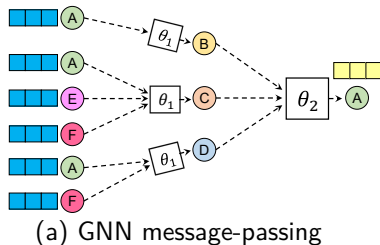


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Are there better alternatives to message-passing for combining node attributes and graph topology for link prediction?

Challenges of GNNs for link prediction

Table: Common real-world datasets for link prediction benchmark.

	#Nodes	#Edges	Avg. degree	Density	Class ratio
CORA	2,708	5,278	3.90	0.14%	1:695
CITESEER	3,327	4,552	2.74	0.08%	1:1216
PUBMED	19,717	44,324	4.50	0.02%	1:4385
PHOTO	7,650	119,081	31.13	0.41%	1:246
COMPUTERS	13,752	245,861	35.76	0.26%	1:385

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Have GNN-based link prediction methods properly addressed the intrinsic class imbalance?

Supervised link prediction evaluation

- ▶ Existing work [13, 14, 15, 16, 17, 18, 19, 20, 21] adopts AUC and AP with *biased testing* (downsampling negative/disconnected pairs), which pictures an **overly optimistic** view of model performance.

Supervised link prediction evaluation

- ▶ Existing work [13, 14, 15, 16, 17, 18, 19, 20, 21] adopts AUC and AP with *biased testing* (downsampling negative/disconnected pairs), which pictures an **overly optimistic** view of model performance.
- ▶ We argue for evaluation under *unbiased testing*, which has been widely applied in unsupervised link prediction [1, 3, 8] and IR.

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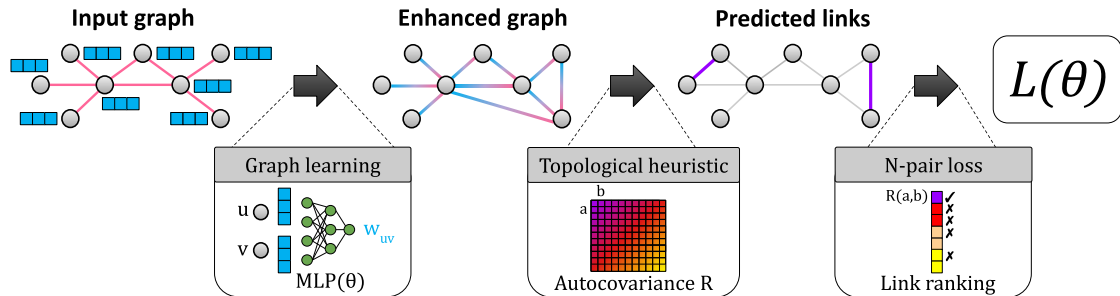
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Supervised link prediction training

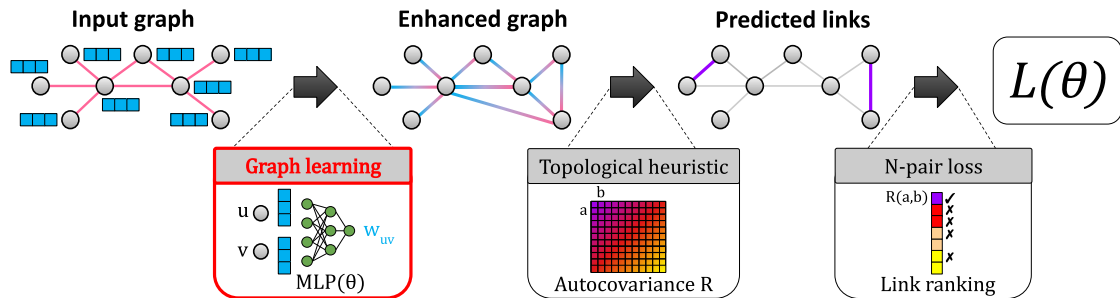
- ▶ Existing work uses binary cross entropy loss with *biased training*.
 - ▶ It **discards** potentially useful evidence from negative pairs.
 - ▶ It induces the model to **overestimate** the probability of positive pairs.

A simpler, faster, and stronger paradigm for link prediction³

- ▶ Explore alternative frameworks to combine topology and attributes
- ▶ Address the problem of class imbalance in training and testing

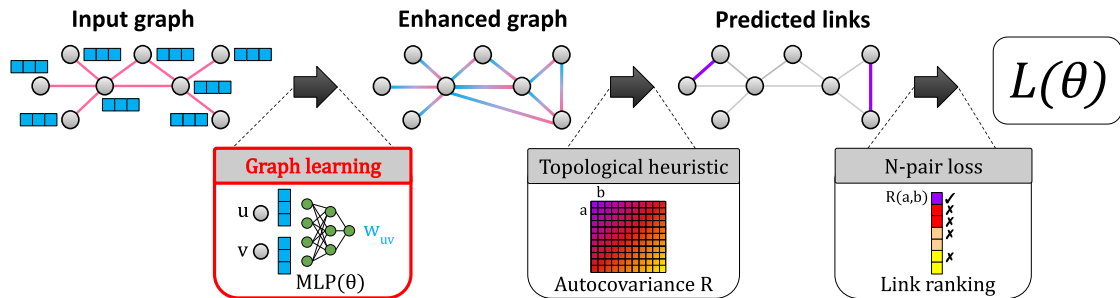


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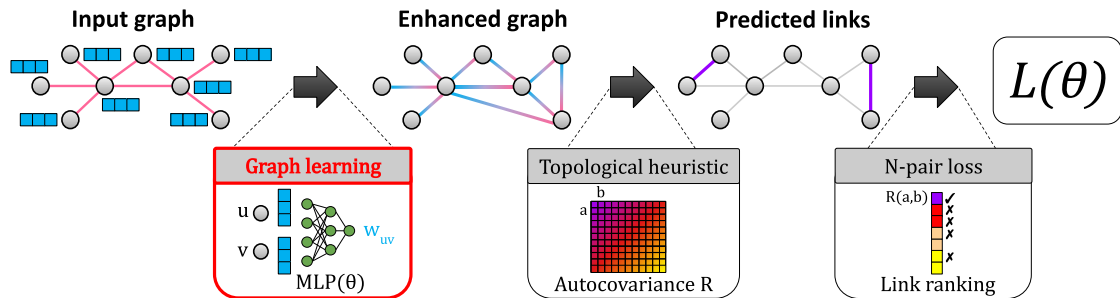
Graph learning

- ▶ Graph augmentation: $\tilde{E} = E + \{(u, v) \mid s(x_u, x_v) > s_\eta\}$



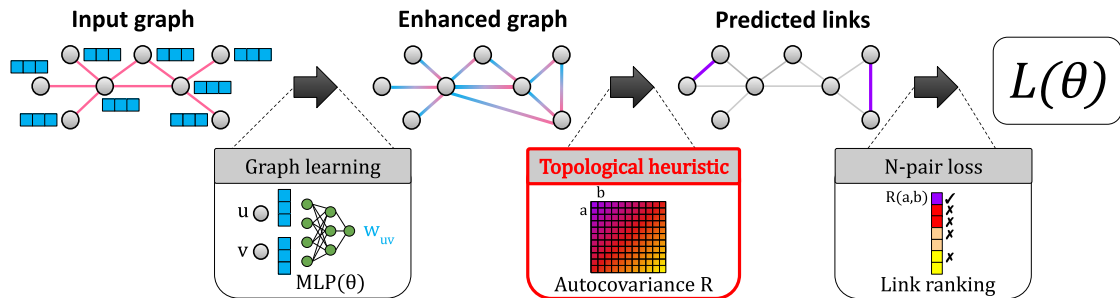
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- ▶ Trained weighting: $w_{uv} = \text{MLP}([x_u + x_v; |x_u - x_v|]; \theta)$



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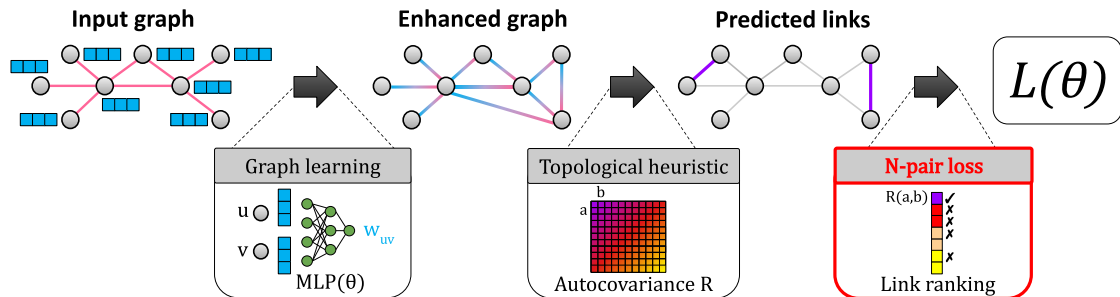
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- ▶ Combined weights: $\tilde{A}_{uv} = \alpha A_{uv} + (1 - \alpha)(\beta w_{uv} + (1 - \beta)s(x_u, x_v))$



Topological heuristic

- ▶ Applying Autocovariance [22, 8] to the enhanced graph $\tilde{A}$:

$$R = \frac{\tilde{D}}{\text{vol}(\tilde{G})} (\tilde{D}^{-1} \tilde{A})^t - \frac{\tilde{d} \tilde{d}^\top}{\text{vol}^2(\tilde{G})}$$



N-pair loss [23]

- ▶ Contrasting **each positive edge** (u, v) with a set of **negative pairs** $N(u, v)$ whose size equals to the class ratio (*unbiased training*):

$$L(\theta) = - \sum_{(u,v) \in E} \log \frac{\exp(R_{uv})}{\exp(R_{uv}) + \sum_{(p,q) \in N(u,v)} \exp(R_{pq})}$$

Table: Link prediction performance comparison (mean $\pm$ std AP). Gelato outperforms the best GNN-based method, Neo-GNN, by **145%** and Autocovariance by **53%**.

* Run only once as each run takes ~ 100 hrs; *** Each run takes > 1000 hrs; OOM: Out Of Memory.

		CORA	CITESEER	PUBMED	PHOTO	COMPUTERS
GNN	GAE	0.27 ± 0.02	0.66 ± 0.11	0.26 ± 0.03	0.28 ± 0.02	0.30 ± 0.02
	SEAL	1.89 ± 0.74	0.91 ± 0.66	***	10.49 ± 0.86	6.84^*
	HGCN	0.82 ± 0.03	0.74 ± 0.10	0.35 ± 0.01	2.11 ± 0.10	2.30 ± 0.14
	LGCN	1.14 ± 0.04	0.86 ± 0.09	0.44 ± 0.01	3.53 ± 0.05	1.96 ± 0.03
	TLC-GNN	0.29 ± 0.09	0.35 ± 0.18	OOM	1.77 ± 0.11	OOM
	Neo-GNN	<u>2.05 ± 0.61</u>	<u>1.61 ± 0.36</u>	<u>1.21 ± 0.14</u>	10.83 ± 1.53	<u>6.75^*</u>
	NBFNet	1.36 ± 0.17	0.77 ± 0.22	***	<u>11.99 ± 1.60</u>	***
	BScNets	0.32 ± 0.08	0.20 ± 0.06	0.22 ± 0.08	2.47 ± 0.18	1.45 ± 0.10
	WalkPool	2.04 ± 0.07	1.39 ± 0.11	1.31^*	OOM	OOM

Table: Link prediction performance comparison (mean $\pm$ std AP). Gelato outperforms the best GNN-based method, Neo-GNN, by **145%** and Autocovariance by **53%**.

* Run only once as each run takes ~ 100 hrs; *** Each run takes >1000 hrs; OOM: Out Of Memory.

		CORA	CITESEER	PUBMED	PHOTO	COMPUTERS
GNN	GAE	0.27 ± 0.02	0.66 ± 0.11	0.26 ± 0.03	0.28 ± 0.02	0.30 ± 0.02
	SEAL	1.89 ± 0.74	0.91 ± 0.66	***	10.49 ± 0.86	6.84^*
	HGCN	0.82 ± 0.03	0.74 ± 0.10	0.35 ± 0.01	2.11 ± 0.10	2.30 ± 0.14
	LGCN	1.14 ± 0.04	0.86 ± 0.09	0.44 ± 0.01	3.53 ± 0.05	1.96 ± 0.03
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Topological Heuristics	CN	1.10 ± 0.00	0.74 ± 0.00	0.36 ± 0.00	7.73 ± 0.00	5.09 ± 0.00
	AA	2.07 ± 0.00	1.24 ± 0.00	0.45 ± 0.00	9.67 ± 0.00	6.52 ± 0.00
	RA	2.02 ± 0.00	1.19 ± 0.00	0.33 ± 0.00	10.77 ± 0.00	7.71 ± 0.00
	AC	<u>2.43 ± 0.00</u>	<u>2.65 ± 0.00</u>	<u>2.50 ± 0.00</u>	<u>16.63 ± 0.00</u>	<u>11.64 ± 0.00</u>

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Attributes + Topology	MLP	0.30 ± 0.05	0.44 ± 0.09	0.14 ± 0.06	1.01 ± 0.26	0.41 ± 0.23
	Cos	0.42 ± 0.00	1.89 ± 0.00	0.07 ± 0.00	0.11 ± 0.00	0.07 ± 0.00
	MLP+AC	3.24 ± 0.03	1.95 ± 0.05	<u>2.61 ± 0.06</u>	<u>15.99 ± 0.21</u>	<u>11.25 ± 0.13</u>
	Cos+AC	<u>3.60 ± 0.00</u>	<u>4.46 ± 0.00</u>	<u>0.51 ± 0.00</u>	10.01 ± 0.00	5.20 ± 0.00
	MLP+Cos+AC	3.39 ± 0.06	4.15 ± 0.14	0.55 ± 0.03	10.88 ± 0.09	5.75 ± 0.11

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Gelato		3.90 ± 0.03	4.55 ± 0.02	2.88 ± 0.09	25.68 ± 0.53	18.77 ± 0.19

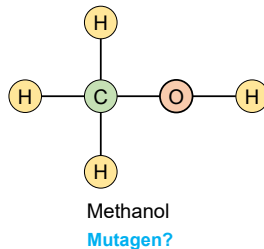
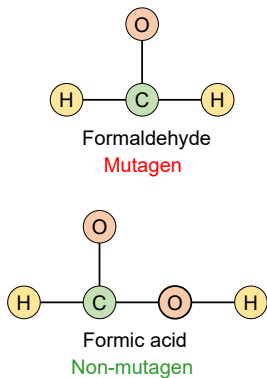
Section summary:

- ▶ We investigated the problem of representation learning for effective link prediction in scale-free, signed, and attributed networks.
- ▶ We scrutinized popular representation learning paradigms (node embedding and GNNs) and proposed novel solutions leveraging random-walk dynamics, social theories, and graph learning.
- ▶ Our methods enable state-of-the-art link prediction performance as verified in various real-world datasets.

Outline

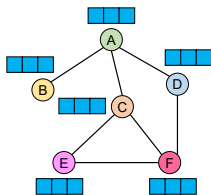
1. Introduction
2. Graph representation learning for link prediction
3. Graph representation learning for graph classification
4. Graph representation learning for tasks on multiscale graphs
5. Conclusion and future work

Graph classification

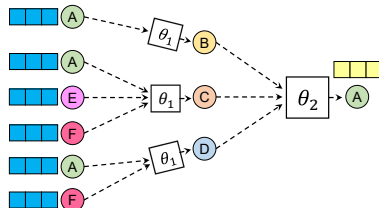


Graph classification: Given a collection of graphs, predict the label of each unknown graph based on known graphs and labels.

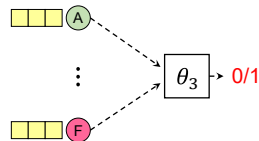
Graph neural networks for graph classification



(a) Input attributed graph



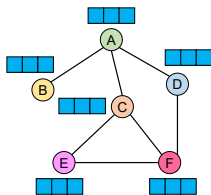
(b) GNN message-passing



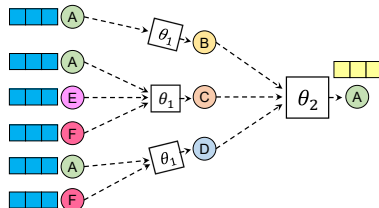
(c) Pooling and classification

- GNNs achieve state-of-the-art performance in graph classification.

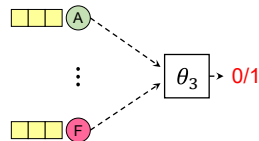
Graph neural networks for graph classification



(a) Input attributed graph



(b) GNN message-passing



(c) Pooling and classification

- GNNs achieve state-of-the-art performance in graph classification.

How can we understand the prediction made by GNNs in high-stakes decision making?

Model understanding and explanation

- ▶ Deep predictive model

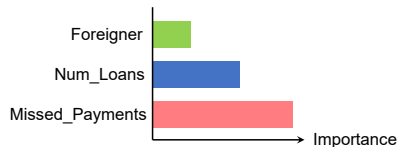
{Foreigner=True, Num_Loans=1,
Missed_Payments=2} $\Rightarrow$ Denied

Model understanding and explanation

- ▶ Deep predictive model

{**Foreigner**=**True**, **Num_Loans**=**1**,
Missed_Payments=**2**} $\Rightarrow$ **Denied**

- ▶ Feature importance [24, 25]

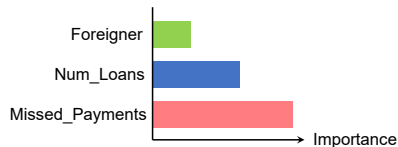


Model understanding and explanation

► Deep predictive model

{**Foreigner**=True, Num_Loans=1,
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► Feature importance [24, 25]



► Local counterfactual [26, 27]

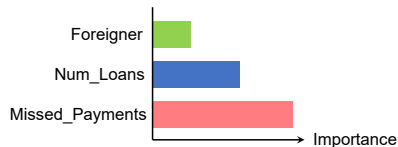
{**Foreigner**=True, Num_Loans=1,
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Model understanding and explanation

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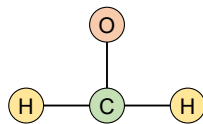
{Foreigner=True, Num_Loans=1,
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► Global counterfactual [28]

*If Foreigner=True and Num_Loans $\geq$ 1
then Missed_Payments $\leq$ 1 for approval*

Understanding GNNs

► Graph classification GNN



Formaldehyde

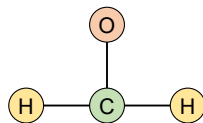


Mutagen

Understanding GNNs

- ▶ Graph classification GNN

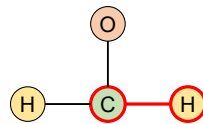
- ▶ Subgraph importance [29, 30]



Formaldehyde



Mutagen

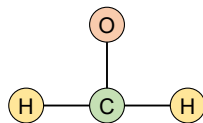


Formaldehyde

Important subgraph

Understanding GNNs

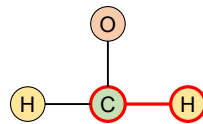
- ▶ Graph classification GNN
- ▶ Subgraph importance [29, 30]
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Formaldehyde

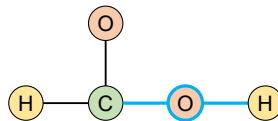


Mutagen



Formaldehyde

Important subgraph



Formic acid



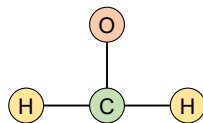
Non-mutagen

Understanding GNNs

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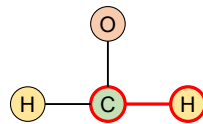
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Formaldehyde

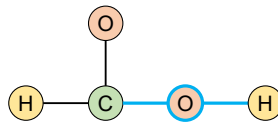


Mutagen



Formaldehyde

Important subgraph



Formic acid



Non-mutagen

Can we generate global counterfactual explanation for GNNs?

First global graph counterfactual explainer for GNNs⁴

⁴**Huang***, Kosan*, et al. Global counterfactual explainer for graph neural networks. WSDM'23.

First global graph counterfactual explainer for GNNs⁴

- ▶ Global recourse: For any undesired graph $G \in \mathcal{G}$ ($\mathbf{GNN}(G) = 0$), the explanation r should provide a recourse: $\mathbf{GNN}(r(G)) = 1$.
- ▶ Interpretable: r should be (much) easier to understand (than $\mathbf{GNN}$).

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Explanation based on counterfactual summary

- ▶ Represent r with a summary set $\mathcal{C}$ of counterfactual graphs.
- ▶ The recourse for G is the minimal cost (distance) summary graph:

$$r_{\mathcal{C}}(G) = \arg \min_{C \in \mathcal{C}} \text{dist}(G, C)$$

where $\text{dist}(\cdot, \cdot)$ is any distance metric between graphs (e.g., GED).

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Quantifying explanation quality

- ▶ Cost: minimize the overall recourse cost for all undesired graphs:

$$\mathbf{cost}(r_{\mathcal{C}}) = \sum_{G \in \mathcal{G}} \min_{C \in \mathcal{C}} \text{dist}(G, C)$$

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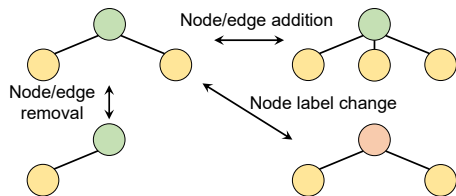
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- ▶ Interpretability: minimize the size of the counterfactual summary:

$$\mathbf{size}(r_{\mathcal{C}}) = |\mathcal{C}|$$

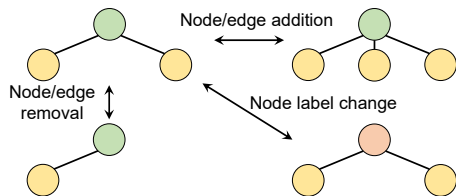
Structuring counterfactual summary search space

- ▶ Graph edit map: A (meta)graph of candidate graphs connected by single graph edits.



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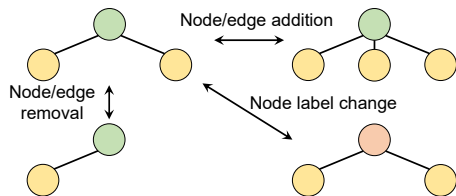


Generating diverse counterfactual summary

- ▶ Vertex-reinforced random-walk: Converges to diverse graphs [33].

Structuring counterfactual summary search space

- ▶ Graph edit map: A (meta)graph of candidate graphs connected by single graph edits.

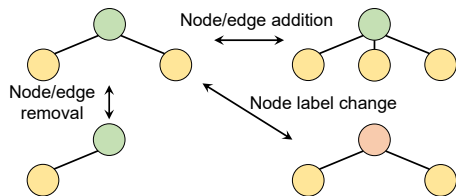


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Generating diverse counterfactual summary

- ▶ Vertex-reinforced random-walk: Converges to diverse graphs [33].
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- ▶ Random teleportation back: Manages the exponential search space.

Comparison with summary of local counterfactuals

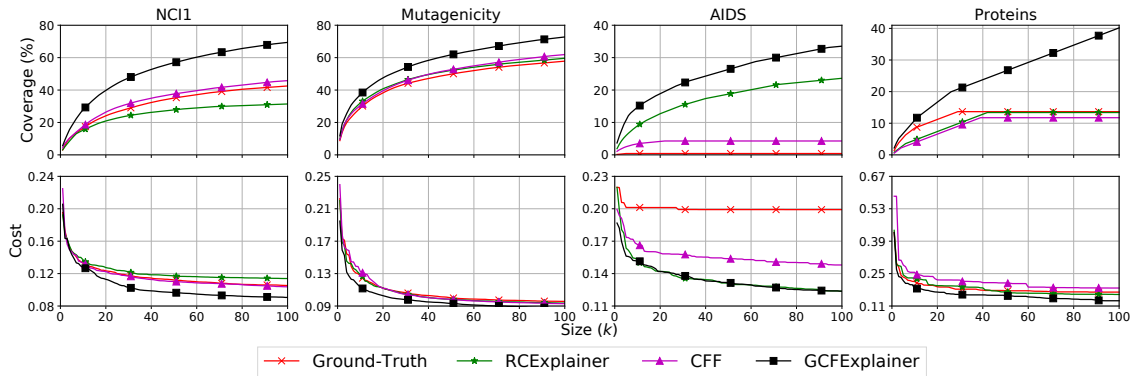


Figure: Global counterfactual quality comparison. GCFExplainer consistently outperforms the baselines with higher coverage and lower cost across different sizes.

Visualizing global counterfactuals

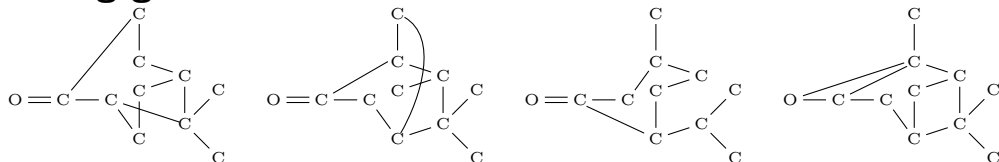


Figure: The global counterfactual explanation graph (third row) presents a high-level recourse rule—changing ketones and ethers into aldehydes—to combat HIV, while the edge removals in local counterfactual examples (second row) are hard to generalize.

Visualizing global counterfactuals

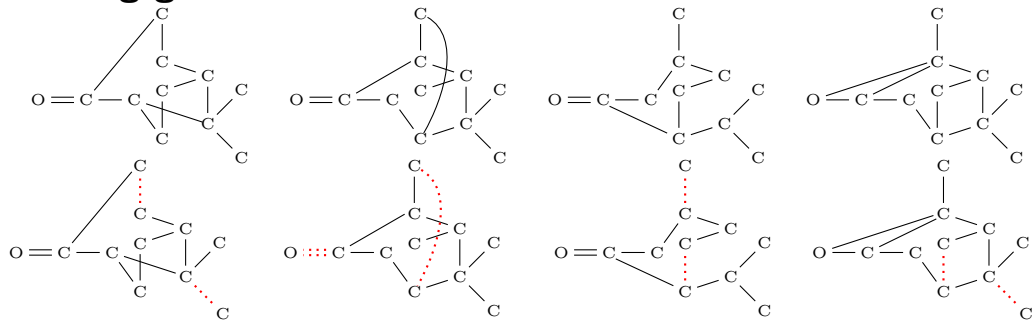


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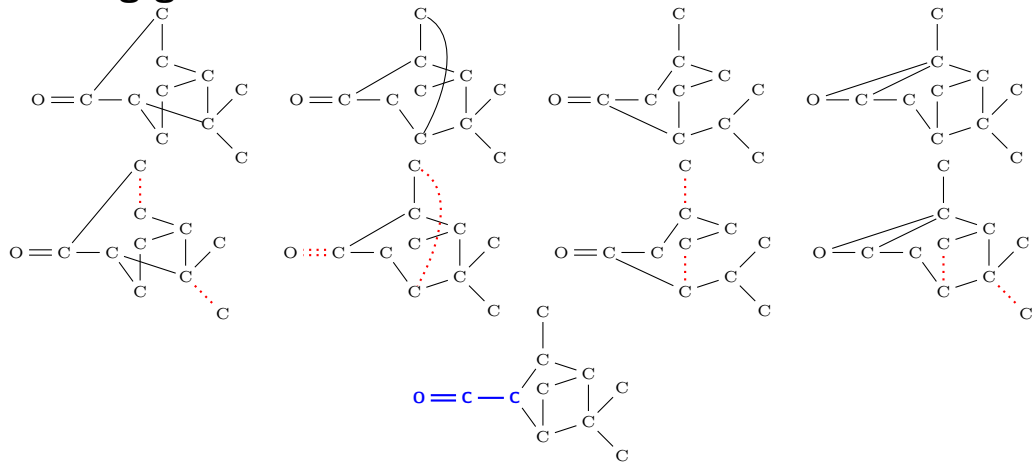


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Section summary:

- ▶ We formulate the novel problem of global counterfactual reasoning and explanation of GNNs for graph classification.
- ▶ We present GCExplainer, the first global counterfactual explainer for GNNs that generates representative counterfactual summaries.
- ▶ We demonstrate the effectiveness and usefulness of GCExplainer in providing high-level recourse for GNN-based graph classification, in the context of drug property prediction and discovery.

Outline

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2. Graph representation learning for link prediction
3. Graph representation learning for graph classification
4. Graph representation learning for tasks on multiscale graphs
5. Conclusion and future work

Multiscale graphs

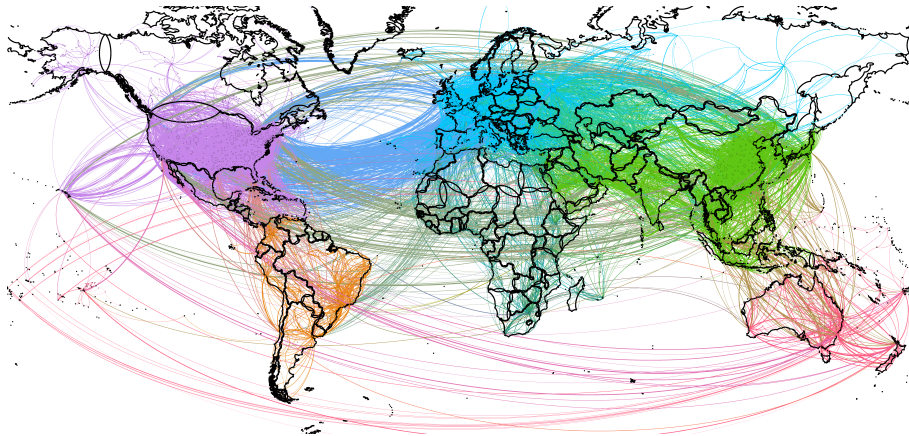


Figure: Global network of airports connected by domestic and international flights. Nodes in multiscale graphs form dense clusters at different structural scales (countries and continents here).

Graph neural diffusion networks⁵

- ▶ Multiscale message-passing as graph diffusions:

$$u^{(K)} = \sum_{k=0}^{K-1} \alpha_k M^k u^{(0)}$$

- ▶ Existing work [34, 35] adopts **fixed** diffusion weights α_k .

⁵Ye, **Huang**, Hong, Singh. Graph convolutional networks meet neural diffusions. Under review.

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- ▶ Existing work [34, 35] adopts **fixed** diffusion weights α_k .
- ▶ We propose to **learn** the diffusion weights directly from data:

$$u^{(K)} = f([u^{(0)}; Mu^{(0)}; \dots; M^{K-1}u^{(0)}]; \theta)$$

- ▶ Key result: The learned weights are **adaptable** to different datasets, leading to better semi-supervised node classification performance.

⁵Ye, **Huang**, Hong, Singh. Graph convolutional networks meet neural diffusions. Under review.

Multiscale community detection⁶

► $R^{AC}(\tau) = \Pi P(\tau) - \pi\pi^\top$ and $R^{PMI}(\tau) = \log(\Pi P(\tau)) - \log(\pi\pi^\top)$

⁶Huang, Kondapaneni, Silva, Singh. Multiscale community detection with pointwise mutual information. Ongoing.

Multiscale community detection⁶

- ▶ $R^{AC}(\tau) = \Pi P(\tau) - \pi \pi^\top$ and $R^{PMI}(\tau) = \log(\Pi P(\tau)) - \log(\pi \pi^\top)$
- ▶ Clustered similarity as a quality function: $r(\mathbf{C}; \tau) = \sum_{i,j \in \mathbf{C}} R_{ij}(\tau)$

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$$r^{AC}(C_1, \dots, C_c; \tau) = \sum_{k=1}^c \sum_{i,j \in C_k} R_{ij}^{AC}(\tau)$$

which assumes a **fixed** scale for each community **specified by the user**.

⁶Huang, Kondapaneni, Silva, Singh. Multiscale community detection with pointwise mutual information. Ongoing.

Multiscale community detection⁶

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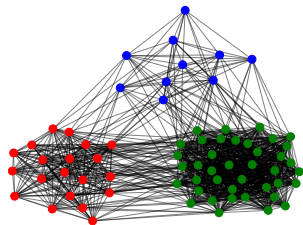
which assumes a **fixed** scale for each community **specified by the user**.

- ▶ We find $\{C_k\}$ along with their **natural scales** $\{\tau_k\}$ based on PMI:

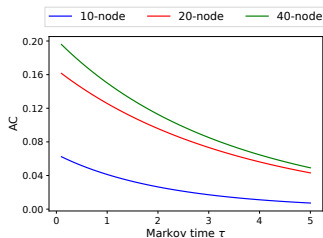
$$r^{PMI}(C_1, \dots, C_c; \tau_1, \dots, \tau_c) = \sum_{k=1}^c \sum_{i,j \in C_k} R_{ij}^{PMI}(\tau_k)$$

⁶Huang, Kondapaneni, Silva, Singh. Multiscale community detection with pointwise mutual information. Ongoing.

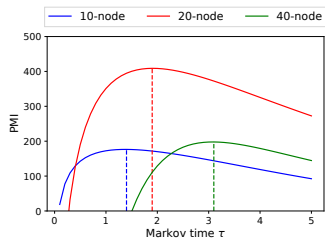
Empirical observation



(a) SBM-(10, 20, 40)



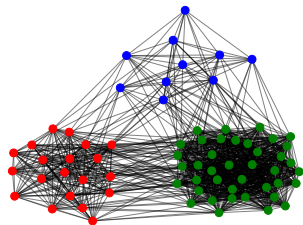
(b) AC



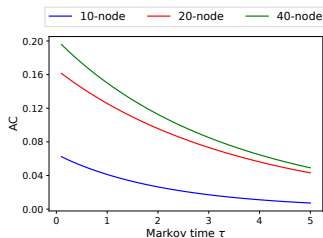
(c) PMI

Figure: Quality functions based on PMI for communities of different sizes reach unique peaks at different τ , revealing their natural scales, while those based on AC cannot.

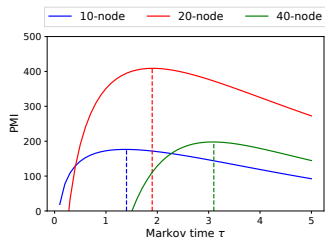
Empirical observation



(a) SBM-(10, 20, 40)



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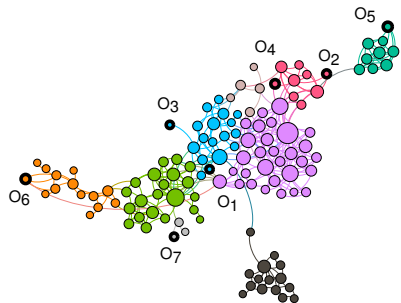
Figure: Quality functions based on PMI for communities of different sizes reach unique peaks at different τ , revealing their natural scales, while those based on AC cannot.

Theoretical analysis

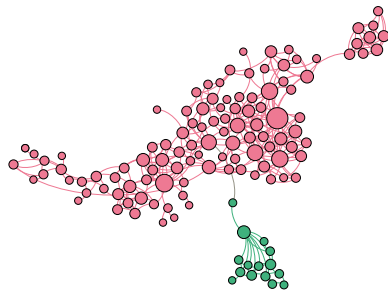
- ▶ We have shown that $r^{AC}(C; \tau)$ monotonously decreases with τ .
- ▶ For $r^{PMI}(C; \tau)$, monotonicity has only been proved for some cases.

Multiscale anomaly detection⁷

- Existing work focuses on **node-level** anomaly within a particular context [36, 37, 38, 39] or multiscale contexts [40, 41].



(a) Node-level anomalies



(b) Subgraph-level anomalies

Figure: Real-world graphs have anomalies of different scales. Figures from [40].

⁷Arriola, Kosan, **Huang**, Singh. Multiscale anomaly detection with graph autoencoders. Ongoing.

Spectral energy distribution patterns for anomalies

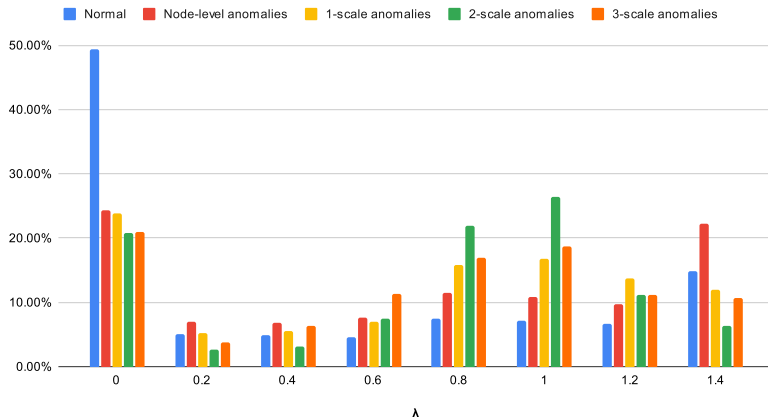
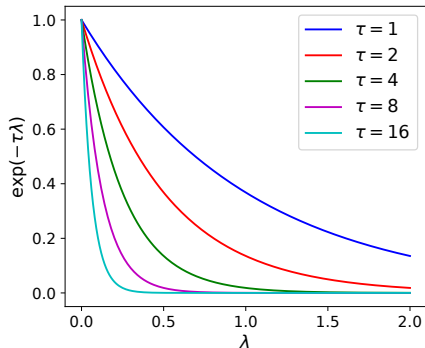
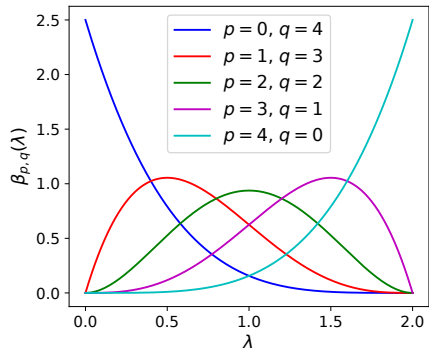


Figure: Compared to normal nodes, the spectral energy distributions of anomalous elements concentrate more on the high-frequency regions. Further, the smaller the scale of the anomalies, the higher the frequency bands they dominate.

Spectral localized Beta Wavelet GNNs



(a) Heat kernels



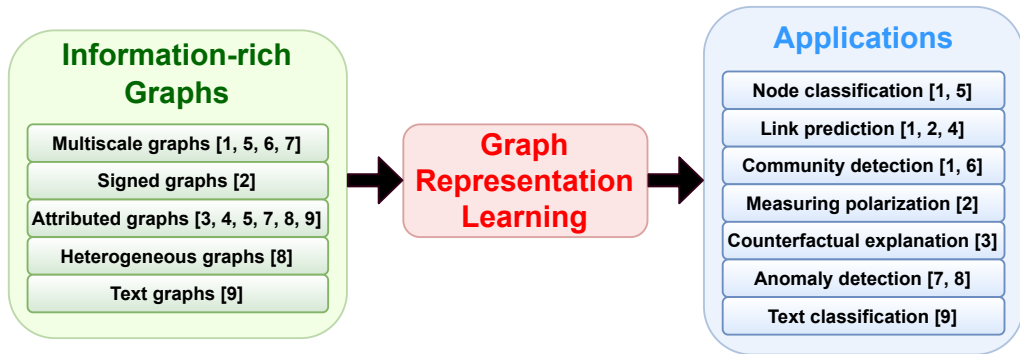
(b) Beta kernels

Figure: Spectral property comparison between heat kernels and Beta kernels. Beta kernels contain different band-pass filters that facilitate multiscale anomaly detection.

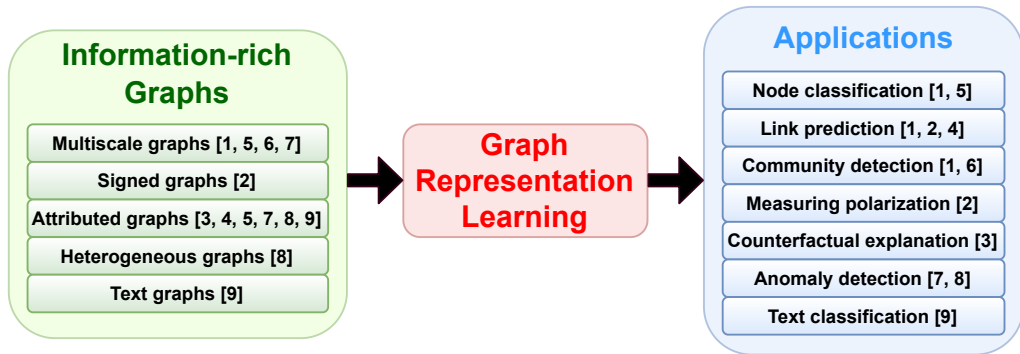
Outline

1. Introduction
2. Graph representation learning for link prediction
3. Graph representation learning for graph classification
4. Graph representation learning for tasks on multiscale graphs
5. Conclusion and future work

This dissertation demonstrates the importance of accounting for the interplay between the rich graph information and downstream task properties in graph representation learning.



This dissertation demonstrates the importance of accounting for the interplay between the rich graph information and downstream task properties in graph representation learning.



- ▶ Can we develop a universal architecture for different graphs & tasks?
- ▶ Can we understand/explain the model in the embedding space?

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My family: *I miss them and this dissertation is dedicated to them.*

Learning Representations for Information-rich Graphs

PhD Defense

Zexi Huang

Committee: Ambuj Singh (Chair), Yu-Xiang Wang, Xifeng Yan

Department of Computer Science, University of California, Santa Barbara

April 5, 2023

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