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Summary Sheet

Parcel with Wings: A Zero-one Programming Model to Manage Logistic Centers for Online Retailers

Summary

Information Age has marked the prime of digital industries, especially online retailers. For an electonic commerce company to thrive, customer satisfaction based on delivery time and price is key.

In this report, we achieve both objectives by manipulation of country-wide warehouse locations, which is presented by a zero-one programming model. Specificly, we accomplish the following:

- **Extracting key features:** Instead of troubling ourselves with exact geographical coordinates of possible locations, we view warehouse as a status for each state, and thus discretize the original problem to an easy-to-handle form. One-day shipping guarantee is then implemented in a convenient manner.
- **Optimizing for the best:** We define several independent criteria for optimazation, including number of warehouses, nearby digital shopper density and total tax liablity of customers. In particular, we introduce Digital Shopper Indicator to weight different states in addition to population and other factors.
- **A status ranking system:** Apart from providing optimal solution for each objective, we introduce a ranking system to serve as a handy reference under changing situations.

Ultimately, we illustrate that our model performs in an easy-to-implement, computationally friendly manner through a case study based on a recreational equippment company in the United States.

Contents

1 Introduction

1.1 Overview

Since the dawn of the Information Age, the digital industry has reshaped people's lifestyles to an astonishing extent, mainly by spanning over its influence on how manufacturing throughout and the service sector operates in an efficient and convenient way [1]. One of the most potent demonstrations is online shopping, where several business giants have risen: Amazon, Alibaba, to mention a few.

Figure 1: Warehouse Logistics of JD [2]. Red dots represent JD logistic centers at Shenyang, Beijing, Shanghai, Chengdu, Wuhan and Guangzhou (from top to bottom, from left to right, respectively). Blue arrows indicate the coverage of these logistic centers.

For an electronic commerce company to thrive, customer satisfaction is the key. With strict quality standards enforced, competitors then consider earning their customers' heart by reducing delivery time and lowing prices. For example, JD, a very successful online retailer in China when it comes to meeting customers' urgent demands, launched its speedy '211 Program' on Mar 31, 2010, which provides same day delivery for orders submitted prior to 11 A.M. and next day delivery (before 3 P.M.) for orders submitted before 11 P.M. based on its newly built warehouses distributed around China (see Figure 1) [3]. The program turned out to an enormous triumph marked by 150% increase in turnover (approximately 10 billion RMB, \$1.47 billion) in a single year [4]. However, it seems that still more digital shoppers don't bother to wait for a longer time if they can enjoy a discounted price, since Tmall, the biggest online retailer in China with market share 2 times more than JD (61.64% vs 18.58%) [5], gains its dominance simply by providing a generally lower price.

In this report, we achieve both objectives (quick delivery and low price) by manipulation of country-wide logistics, that is, finding best locations of logistic centers (warehouses) that not only guarantee one-day ground shipping through certain package delivery company, but also reduce tax liability of customers as much as possible.

1.2 Nomenclature

The nomenclature used in this report is listed as follows:

Symbol	**Definition**
N	Number of states in target country
x_i	Warehouse status in ith state, 1 for a warehouse is located, and 0 otherwise
c_{ij}	One-day reachability from the capital of ith state to jth state, 1 for reachable, and 0 otherwise
ρ_i	Population density of ith state
p_i	Population of ith state
s_i	Per capita income of ith state
d_i	Digital shopper indicator of ith state, measuring to what extent customers prefer online shopping to offline shopping
t_i	The sales tax rate of ith state for the goods of the industry the company is currently in
$t_i^{'}$	The sales tax rate of ith state for the goods in the new industry that the company wants to dive in
η_1	The proportion of money customers spend on the industry that the company is curretly in.
η_2	The proportion of money customers spend on the new industry that the company wants to dive in.
Z_1	Number of warehouses to be placed, the primary objective function in Part I
$Z_1^{'}$	The secondary objective function in Part I, representing the mean digital shopper densities around each warehouse
Z_2	The objective function in Part II, representing the total tax liability of customers across target country
Z_3	The objective function in Part III, representing the combination of tax liability from both general goods and goods in the new industry
λ	The weight for general industry and the new industry in Part III

1.3 Assumptions

The reliability of our models depends on certain key, simplifying assumptions, which are listed below:

- Within each state, the delivery time is no more than one day.
- There is either one warehouse located in its capital or none in a state.
- If the capital of a state can be reached in one day by a warehouse, then the whole state is considered reachable, and otherwise unreachable.
- The proportion of money spent on same catagory of goods is constant in target country.

The tax policy used is also defined arbitrarily in order to present consistent results in this report. This, however, can be varied to accommodate a specific target country:

- Only state's tax is considered.
- Any online order delivered to a location within a state where a warehouse is located will have that state's tax added to the order cost.
- Any online order delivered to a location outside a state where a warehouse is located will not be taxed.

2 Model Theory

2.1 Zero-one Programming Model

First of all, we introduce the general idea of how we formulate the problem.

We consider target country as a collection of N geographical states (or its counterparts, e.g., provinces in China). It is first assusmed that within each state, the delivery time is no more than one day while cross-state transportation could cost up to several days depending on relative distance. This assumption holds for most states in most countries with fair infrastructure and package delivery company. As is the case of JD, the warehouse in state are often located in its capital with several ecomical considerations, for example, to serve most potential customers and to utilize most convenient transportation facilities. Thus, what we need to determine is whether, in each state, a warehouse should be located in its capital. For N states, denote x_i as the zero-one decisional variable indicating warehouse status in ith state, that is

$$x_i = \begin{cases} 1, \text{a warehouse is located in } i\text{th state} \\ 0, \text{no warehouse is located in } i\text{th state} \end{cases}, 1 \leqslant i \leqslant N \tag{1}$$

Then, we move on to analyze the delivery network among states. For a certain package delivery company providing on-schedule delivery service (e.g. United Parcel Service (UPS)), the maximum deliver time given departure and destination is readily available on its service terms. Therefore, for a determined delivery company, we can easily obtain all the possible one-day routes between states. However, it is noteworthy that in some cases, only part of state can be covered by one-day travel from the capital of another state. For example, Figure 2 shows the area and respective transit days for UPS from Indianapolis, capital of Indiana. As we can see, Illinois, Missouri and Kentucky are only partially covered by one-transit-day area. To deal with the ambiguity, we assume that if the one-transit-day area covers the capital of another state, then the whole state is considered reachable in one day, otherwise unreachable. This assumption lies in the fact that generally a capital is home to a large part of the state's population and it is convenient to transport required packages to other part of state. Then in Figure 2, both Illinois and Kentucky is considered reachable while Missouri is not. With problem of ambiguity solved, we denote c_{ij} as the status of one-day reachability from the capital of ith state to jth state, that is

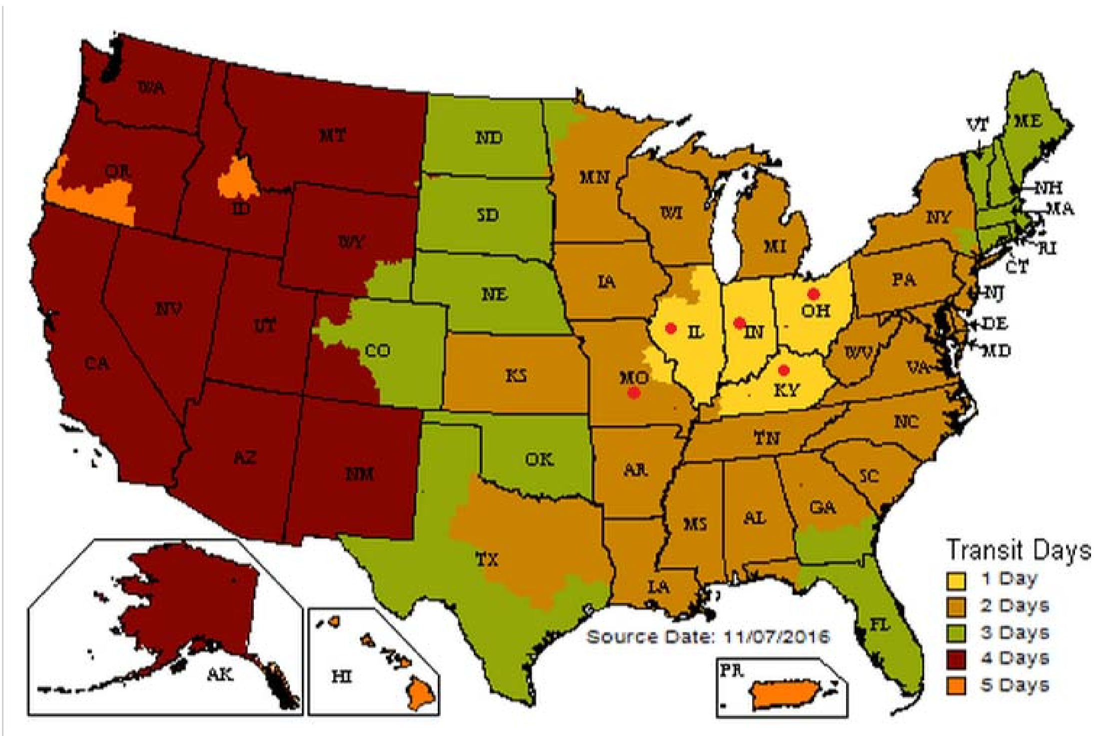

Figure 2: Outbound view from Indianapolis for UPS. Red dot depicts repective capital for each state.

$$c_{ij} = \begin{cases} 1, j\text{th state can be reached from the capital of } i\text{th state in one day} \\ 0, j\text{th state can't be reached from the capital of } i\text{th state in one day} \end{cases} \tag{2}$$
$$1 \leqslant i, j \leqslant N$$

Note that according to our first assumption, it's obvious that $c_{ii} = 1, 1 \leqslant i \leqslant N$.

Now, if we want to establish a logistic network in target country to provide one-day ground shipping guarantee, the requirement can be stated as, 'for any state, it is one-day reachable from at least one capital where a warehouse is located'. This leads to the following constrains:

$$\bigvee_{j=1}^{N}(x_j \wedge c_{ji}) = 1, \text{ for } 1 \leqslant i \leqslant N \tag{3}$$

or equivalently,

$$\sum_{j=1}^{N} x_j c_{ji} \geqslant 1, \text{ for } 1 \leqslant i \leqslant N \tag{4}$$

Since the decision variables and constrains are all set, we have almost successfully transformed the original problem into a zero-one programming model. All we left to do is to determine what objective to accomplish. For the following subsections, we detail 3 considerations.

2.2 Part I: Minimizing Number of Warehouses

The first objective we discuss here is to minimize number of warehouses. The rationale is to reduce initial investment in placing new warehouses as well as maintenance cost that follows. Under such consideration, the objective can be expressed as

$$\text{Minimize } Z_1 = \sum_{i=1}^{N} x_i \tag{5}$$

where Z_1 represents the total number of warehouses to be placed.

Equation 1, Equation 4 and Equation 5 together make up a complete zero-one programming problem. Taking advantage of computer-based optimization tools (e,g, Lingo), the optimal objective value Z_1^* and respective optimal solution(s) can be easily determined.

However, in our case, it is expected that multiple optimal solutions exist as there may be serveral choices of locations for a given number of warehouses. To obtain the exact locations of optimal placement, we introduce an extra selection criterion to choose the best out of all the optimal solutions.

As summarized in [6], state population density is an important factor as to choice of warehouse location. Here, we take a step further by considering the population density of potential digital shoppers in a state. Since the exact figure is almost impossible to count, we introduce a state-dependent DSI (**D**igital **S**hopper **I**ndicator) d_i for estimation of ith state. Its rationale and definition are introduced below.

The reasons why people prefer online shopping to offline shopping can be complicated and vary from person to person, but convenience and economy are common. Convenience is no more a problem since one-day shipping is guaranteed, but when customers find goods online are cheaper than offline, it's another story. For a specific goods online, its price across the country is constant, while offline prices can be various. For example, Figure 3 shows the real value of $100 in each state in the United States. For people in California whose $100 can only purchase stuff that would cost $88.97 at the national average price level offline, it is natural to resort to online resources. Thus, we define DSI as the real value of certain amount of money online (which is equal that amount) and that offline (which is shown in Figure 3, as for the United States), indicating to what extent customers prefer online shopping to offline shopping. For example, for California, DSI is $100/88.97 \approx 1.124 > 1$, indicating Californians are more likely to purchase online goods than national average.

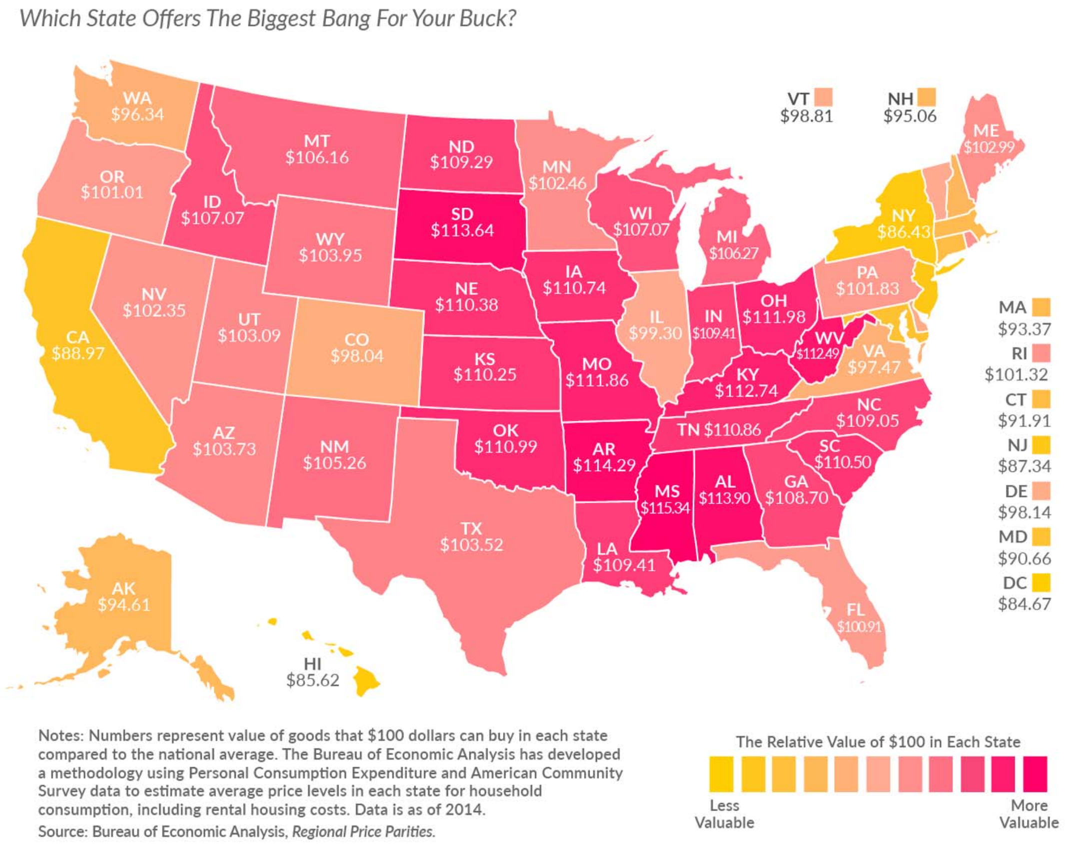

Figure 3: The real value of \$100 in each state in the United States in 2014 [7]. Purple indicates high value of \$100 while yellow indicates the opposite.

Now, we could use the product of DSI d_i and population density ρ_i for ith state, that is approximately proportional to actual digital shopper density, to represent it. Then, using this product as the weight for each warehouse while keeping the number of warehouses as its optimal value, we maximize the secondary objective $Z_1^{'}$, which is approximately proportion to the mean digital shopper densities around each warehouse. This leads to

$$\text{Maximize } Z_1^{'} = \sum_{i=1}^{N} \rho_i d_i x_i \tag{6}$$

and

$$\sum_{i=1}^{N} x_i = Z_1^{*} \tag{7}$$

This objective function is expected to yield only one optimal solution considering both the weights and the objective are continuously valued. And thus, the optimal placement is determined for Part I.

2.3 Part II: Minimizing Tax Liability of Customers

As mentioned in Subsection 1.1, price is another determining factor for an online retailer to thrive. The detailed discussion of price is very complicated and thus excluded in this report. However, the tax liability of customers is much easier to analyze. Often, tax is often added to the origin price of the good, which increases financial burden of customers, compromising the company's competitiveness. By taking possible advantange of tax policy, we may help customers to avoid tax. A special tax policy is discussed here as follows:

- Only state's tax is considered.
- Any online order delivered to a location within a state where a warehouse is located will have that state's tax added to the order cost.
- Any online order delivered to a location outside a state where a warehouse is located will not be taxed.

Since DSI indicates the willingness a customer would choose online shopping instead of offline, we use the product of DSI d_i and population p_i to represent the total number of online customers in ith state, just like the discussion of the population density of online customers before. To evaluate how much money is spent for a single online customer for a certain period of time, we use average per capita income s_i as the indicator. Then, to minimize the total tax that customers have to pay, we have

$$\text{Minimize } Z_2 = \sum_{i=1}^{N} d_i p_i s_i t_i x_i \tag{8}$$

where Z_2 represents the total tax liability of customers across target country and t_i is the tax rate in ith state.

Sovling Equation 1, Equation 4 and Equation 8 we can have optimal solution for Part II.

2.4 Part III: Diving into New Industry

For an expanding company, diversification is as important as intergration and concentration. If an online retailer wants to grow, increasing goods variety is a widely-applied choice. For example, Amazon started as an online bookstore, later diversifying to sell electronics, apprarel, furniture, etc. However, diving into new industry means fierce competition with experienced pioneers.

In this report, we consider earning share of cake also by taking advantage of tax polices. As an instance, in the United States, several states such as Vermont [8] and

New Jersey [9] exempt sales tax of most type of clothing. Locating warehouses in such states can relieve tax burdens of customers and help newcomers earn a place. Consider the tax rate in ith state for the goods in new industry is $t_i^{'}$. Then, we optimize a linear combination of general tax and tax for new industry to determine a hybrid strategy for warehouse location, that is

$$\text{Minimize } Z_3 = (1-\lambda)\eta_1 \sum_{i=1}^{N} d_i p_i s_i t_i x_i + \lambda \eta_2 \sum_{i=1}^{N} d_i p_i s_i t_i^{'} x_i \tag{9}$$

where λ is the weight of the two terms and η_1 is the proportion of money for customers spent on the goods the company is currently in, while η_2 is that of the new industry. Both η_1 and η_2 are assumed to be constant in for a certain country and are readily available for most industries in most countries.

By tuning the weight λ, the company can find optimal solutions under different circumstances.

3 Model Implementation and Results

We then implement our model and discuss the results, based on a case study.

The company we choose is a online retailer that sells recreation equippment online and deliever it to customers with UPS in contineтal United States. Its headquaters is located in New Hampshire and now seeks expansion by placing new warehouses in the whole country and providing one-day ground shipping service. It also decides to dive into clothing and apprarel selling, a new industry for it. Prior to implemention, the necessary data is collected or computed: c_{ij} [10], ρ_i [11], p_i [12], s_i [13], d_i [7], t_i and $t_i^{'}$ [14], η_1 and η_2 [15].

3.1 Part I: Minimum Number of Warehouses with a Ranking System

The problem of the primary objective Z_1 yields mutiple optimal solutions as expected, with optimal objective value $Z_1^* = 19$, that is, there should be at least 19 warehouses (including the main warehouse in New Hampshire) placed to guarantee one-day shipping. It is noteworthy that although there are many optimal solutions, for each state, its warehouse status can be divided into four catagories, or rankes, as shown in Table 1. They are explained below:

- **'X':** Whether a warehouse is located in this state doesn't impact the optimal objective value. This means the warehouse in this state can be replaced by a warehouse in another, if required.
- **'1':** A warehouse must be located in this state, otherwise one-day shipping guarantee will be broken. That's because this state can't be reached by any other states in one-day.
- **'$1*$':** If the warehouse in the state is replaced, at least two other states should be placed warehouses for compensation. This state is viewed as key logistic centers as it can reach several other states in one day and also its function is irreplacable.

- **'$0*$':** No warehouse should be located in this state, and adding one would do nothing about one-day ground guarantee but increase cost. Most of these states are near logistic centers.

Table 1: States divided into four ranks depending on their warehouse status.

Rank	Number of states	Example of states
X	12	Colorado, Wyoming, New Hampshire, Oregon
1	10	New Mexico, Texas, Montana, California
$1*$	3	Kansas, Illinois, Alabama
$0*$	23	Missouri, Indiana, Vermont, Iowa

The application of the ranking system is to find alternative solutions when extra requirements are indicated. For example, in Figure 4, if a warehouse is required to be excluded in one of the four states, Colorada, Kansas, New Mexico, Texas, by referring to Table 1, we can quickly find out Colorado should be ruled out since only it belongs to catagary 'X'. In fact, as may be expected by the reader, it should be replaced Wyoming, another state belonging to 'X'.

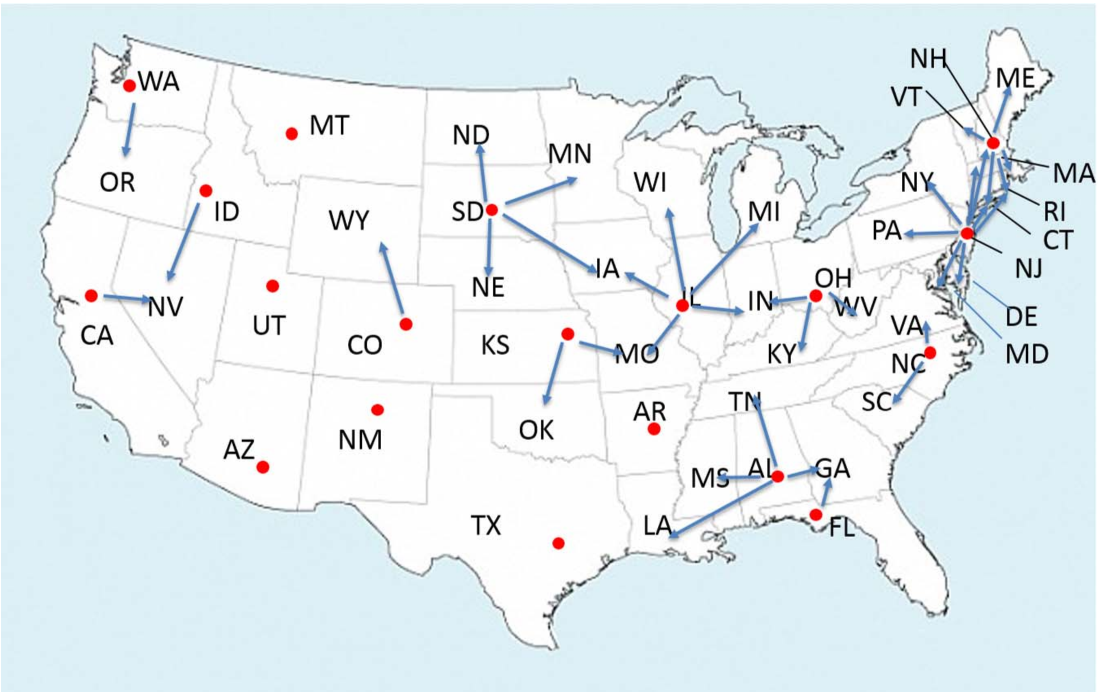

Figure 4: Graphic illustration of the optimal solution for the secondary objective function in Part I (19 warehouses including New Hampshire). Red dots represents warehouse locations. Blue arrows indicate the coverage of these warehouses.

After adding state digital shopper density as a criterion, the secondary objective Z_1' is optimized. Only one optimal solution is found and it is shown in Figure 4. Compare it with Figure 5, the population density map of the United States, we summarize two key findings:

- **The density preference is respected.** Three main density peaks, Los Angeles in

Canifornia, Chicago in Illinois and New York City with New Jersey as its neighbour, are all chosen as warehouse locations.

- **The one-day ground shipping is top priority.** Though low in population density (and thus digital shopper density), states like Montana, Utah and Idaho are still chosen as warehouse locations.

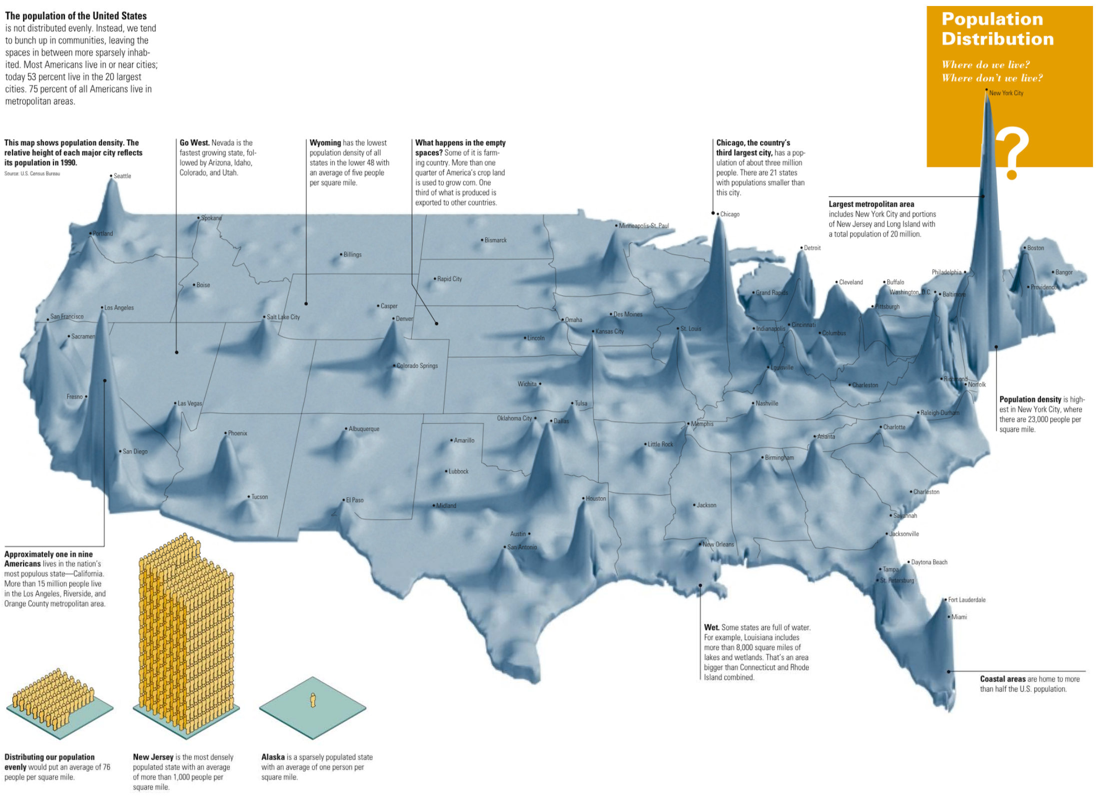

Figure 5: Population density map in the United States [16]. High values indicate large population densities.

3.2 Part II: Minimum Tax Liability with Trade-off

After changing criterion from nearby digital shopper density to tax liability, the warehouse locations specified in Figure 4 seems not that ideal. For example, the warehouse in Trenton, New Jersey, the one also mentioned in Part I as it's near New York Population Peaks, would render a heave tax burden to its customers in state since its sale tax rate is 7.00%, only second to Canifornia. However, its neighbour state, Delaware, with one-day coverage almost the same as New Jersey, exempts all kinds of sales tax. It's also the same case for Washington and Oregon, as Washington adopts a high rate 6.50% while Oregon is a tax-free state.

The optimal solution of Part II (Z_2) confirms our previous ideas and also reveals other insights. The solution is illustrated in Figure 6, with a total of 20 warehouses. We summarize these insights:

- **Make the most of tax-free states.** All four tax-free states in continent United States, New Hampshire, Delaware, Oregon and Montana are chosen as warehouse locations.
- **Trade-off between warehouse numbers and tax.** Illinois, viewed as logistic centers as catagorized as '1∗' in Table 1, is no longer chosen as warehouse location. This directly leads to the optimal solution indicating 20 warehouses placed instead of 19 as in Part I. However, consider the relative high tax rate of Illinois, its large population (see Figure 5) and its above-average DSI (see Figure 3), building two new warehouses elsewhere is rational.
- **One-day shipping must be respected.** Consider Canifornia, the state collects the most sales tax in the whole country with its largest population and highest tax rate, is still a warehouse location. After all, no other states could reach the whole Canifornia in one day.

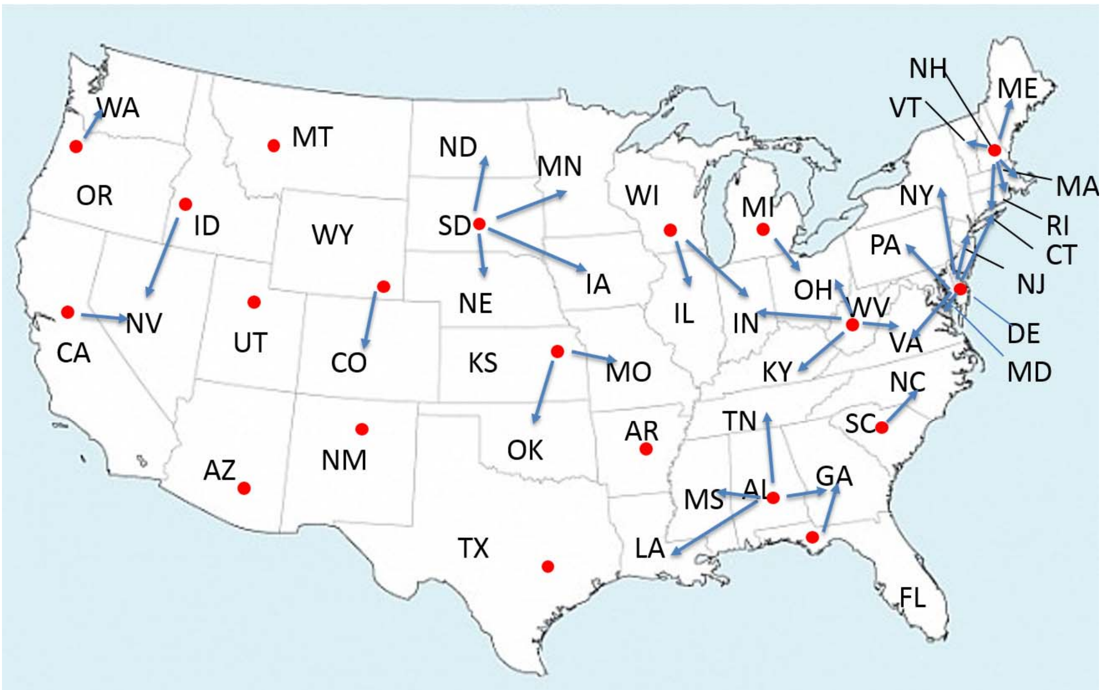

Figure 6: Graphic illustration of the optimal solution for the objective function in Part II (20 warehouses including New Hampshire). Red dots represents warehouse locations. Blue arrows indicate the coverage of these warehouses.

3.3 Part III: New Field with No change

Generally speaking, with different tax rates, we can manipulate the locations of our warehouses to achieve different objectives, like entering a new field. However, in this case study, also to our surprise, the optimal solution for minimizing apparel tax remain the same as that in Part II for any weight $\lambda < 1$. If $\lambda = 1$, there are mutiple optimal solutions, but their only difference are some dummy warehouses in apparel tax-free states. The reasons are explained as follows.

Under current tax policy in the United States, only seven states that exempt sales tax for apparel and collect recreational goods, which means they are the only ones with different weights between Part II and Part III. Now, we could also develop a similar ranking systems of these states for Part II, as shown in Table 2. We note that all of them are catagorized in '0∗', meaning that for each of them, after tuning its tax rate to zero in Part II and locating a warehouse there, the optimal solution will not change. That is, they can't help improve the one-day ground shipping services when we need to keep tax liability as low as possible.

Table 2: Ranks and tax rates for recreational goods of seven states that exempt tax on apparel.

States	Rank	Value in Part II	Tax Rate for Recreational Goods
Massachusetts	0∗	0	6.25%
Minnesota	0∗	0	6.88%
New Jeresy	0∗	0	7.00%
New York	0∗	0	4.00%
Pennsylvania	0∗	0	6.00%
Rhode Island	0∗	0	7.00%
Vermont	0∗	0	6.00%

Thus, we conclude that in this case, there is no need for any changes about number or locations of warehouses, and the locations in Figure 6 are optimal for both parts.

4 Final Remarks

4.1 Strengths and weaknesses

We formulate the problem as a zero-one programming model and it is purposely designed to be customizable and applicable to a variety of scenarios. This leads to a set of strengths and weakeness. Strenths:

- **Applies widely.** This model is capable of being used in most countries in the world by incorporating country-dependent factors like population and per capita income. In addition, different package delivery companys can be easily considered and different objectives can be optimized.
- **Efficient computation.** Thanks to the well developped Branch and Bound (B-B) algorithm for zero-one progromming, this model is much more efficient than real-valued problems.
- **Adopts to changing situaion.** Apart from an optimal solution for different objective, a ranking system is also developped as quick reference for analysis if situation changes and extra constrains are provided.

Weakness:

- **Relies on several simplifying assumptions.** In order to precisely model a real world scenario, these assumptions would need to be removed and incorporate into the model.
- **Limited consideration.** We only discuss several objectives and negelect some other factors, such as the cost of each warehouse.

4.2 Future Development

Most of the issues arise from some assumtions are over-simplyfied and unique factors of the scenario are overlooked. In the future, we would further develop and optimize our model in the following ways:

- Consider the situation when a full state can't be reached from its capital in one day. So that multiple houses are needed in a single state.
- Modify the definition of one-day reachability between states so that it is more realistic.
- Account for topographic factor and take transport routes into consideration.
- Rework the part for DSI and define it over more factors.
- Use state-dependent proportions of money spent on certain types of goods instead of a national constant.
- Consider the cost (both initial and long-term) of setting a warehouse in a specific state.
- Consider a more realistic tax systems, adding city's tax and county's tax to the model.

As for the case study part, we would like to

- Find a more realistic case to better illustrate the process.
- Consider cases in different countries with different types of delivery.

4.3 Conclusion

We model the optimal expansion strategy for an online retailer through placing warehouses countrywide and providing one-day ground shipping services. Firstly, a zero-one programming model is formulated as we extract key features of the real-life problem. After that, several critical objectives for a company to thrive, including warehouses avaliability to potential customers and prices (represented by customer tax liability) are optimized, during which Digital Shopper Indicator is introduced to estimate proportion of online customers. Finally, we illustrate our model through a real-life situation and develop a practical ranking system for changing situations. Overall, our model provides a easy-to-implement and computationally friendly approach to quickly analyze the optimal locations of warehouses for expansion of an online retailer.

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